

MR2118769 (2005i:11013) 11A55 (11D09)

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Necessary and sufficient conditions for the central norm to equal 2^h in the simple continued fraction expansion of $\sqrt{2^h c}$ for any odd $c > 1$. (English summary)

Canad. Math. Bull. **48** (2005), *no. 1*, 121–132.

In his previous paper [JP J. Algebra Number Theory Appl. **4** (2004), no. 1, 159–207; [MR2049695 \(2005a:11031\)](#)], the author gave necessary and sufficient conditions for the parity of the period length of the simple continued fraction expansion of \sqrt{D} for any nonsquare $D > 1$ in terms of solvability of the Diophantine equations $ax^2 - by^2 = \pm 1, \pm 2$, which implies that the central norm is equal to 2 in such an expansion if and only if there is a solution to the Diophantine equations $x^2 - Dy^2 = \pm 2$. In this paper, the author explicitly identifies those D for which this holds in terms of congruence conditions on the prime divisors of D , and parity conditions on certain numerators of convergents as well as the period length itself. Therefore he gives a complete description of those $D = 2^h c$ ($c > 1$ odd and possibly a square when h is odd) with central norm equal to 2^h .

Reviewed by [Pingzhi Yuan](#)

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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