MR2139972 (2005m:11042) 11D09 (11R11 11R29)

Mollin, R. A. (3-CALG-MS)

Eisenstein equations and central norms. (English summary)

C. R. Math. Acad. Sci. Soc. R. Can. **27** (2005), *no. 1,* 20–24.

Lagrange proved that if p is an odd prime, and (x_0, y_0) is the fundamental solution of the Pell equation $x^2 - Dy^2 = 1$, then $x_0 \equiv 1 \pmod p$ if and only if $p \equiv 7 \pmod 8$. In this paper, the following even analogues of the above result applied to the Eisenstein equation $x^2 - Dy^2 = -4$, with $\gcd(x, y) = 1$, are obtained.

Theorem 2: Let $D = 4c$, where c is an odd integer that is not a perfect square, and $l' = l(\sqrt{2c})$, where $l(\sqrt{D})$ denotes the period length of the simple continued fraction expansion of $\sqrt{D}$. Also assume that (x_0, y_0) is the fundamental solution of the Eisenstein equation $x^2 - Dy^2 = -4$, with $\gcd(x, y) = 1$. Then the following are equivalent:

(i) $A_{l'-1} \equiv 1 \pmod{2c}$.

(ii) $l = l(\sqrt{D})$ is even, $Q_{l/2} = 4$, $l/2$ is odd, l' is even with $Q_{l'/2} = 2$, and $l'/2$ is even.

(iii) The Diophantine equation $x^2 - 2cy^2 = 2$ has a solution.

Reviewed by *Pingzhi Yuan*