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Uniform distribution and the Schur subgroup.

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Let K be a finite abelian extension of the rational field Q , and let A be a central simple algebra over K . Suppose that m is the index of A and that K contains a primitive m th root of unity ε . Then A is said to have uniformly distributed (Hasse) invariants provided that $\text{inv}_{\mathfrak{p}}(A) \equiv s \text{inv}_{\mathfrak{p}} \sigma(A)$ modulo 1 whenever \mathfrak{p} is prime of K and $\sigma \in \text{Gal}(K/Q)$ with $\varepsilon^\sigma = \varepsilon^s$. This paper is concerned with the study of $U(K)$, the subgroup of the Brauer group $B(K)$ consisting of classes containing algebras with uniformly distributed invariants. The subgroup $U(K)$ contains the Schur subgroup $S(K)$, which consists of algebra classes which contain a simple component of a group algebra for a finite group, as was shown by the reviewer and M. M. Schacher [same *J.* **22** (1972), 378–385; MR0302747 (46 #1890)]. The results of this paper provide some information about the relationship between $U(K)$ and $S(K)$. Let p be a prime and let $U(K)_p, S(K)_p$ denote the p -torsion parts of these groups. It is shown that if ζ is the largest p -power root of unity in K and $p \nmid |K:Q(\zeta)|$, then $|U(K)_p: S(K)_p|$ is infinite except in certain (infinitely many) special cases. As a contrast, an example is given to show that $U(K)_p = S(K)_p$ is also possible when $p \nmid |K:Q(\zeta)|$. There are several other results, including the calculation of generators for $U(K)_2$ for certain fields. *Mark Benard*