
MR2188254 (Review)
Mollin, R. A. (3-CALG-MS)
Generalized Lagrange criteria for certain quadratic Diophantine equations. (English summary)
New York J. Math. **11** (2005), 539–545 (*electronic*).
11D09 (11A55 11R11 11R29)

Suppose D is a positive integer that is not a perfect square and that (x_0, y_0) is the fundamental solution to the equation $x^2 - Dy^2 = 4$. This paper characterizes whether the congruences $x_0 \equiv \pm 2 \pmod{D/2}$ hold in terms of the continued fraction expansion of $\sqrt{D}$.

This can be seen as a generalization of Lagrange's criterion for the fundamental solution (x_0, y_0) of the Pell equation $x^2 - py^2 = 1$, where p is an odd prime. This says that $x_0 \equiv 1 \pmod{p}$ if and only if $p \equiv 7 \pmod{8}$.

Various consequences for the continued fraction expansion of $\sqrt{D}$ are also discussed. The paper is concise and the proofs use only the basic properties of continued fractions.

Reviewed by *Johnny Edwards*

[References]

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

1. G. Eisenstein, *Aufgaben*, J. Reine Angew. Math. **27** (1844), 86–88, ERAM 027.0784cj.
2. P. Kaplan and K.S. Williams, *Pell's equations $x^2 - my^2 = -1, -4$, and continued fractions*, J. Number Theory **23** (1986), 169–182, MR0845899 (87g:11035), Zbl 0596.10013. MR0845899 (87g:11035)
3. R.A. Mollin, **Quadratics**, CRC Press Series on Discrete Mathematics and its Applications, CRC Press, Boca Raton, 1996, MR1383823 (97e:11135), Zbl 0858.11001. MR1383823 (97e:11135)

4. R.A. Mollin, *Fundamental number theory with applications*, CRC Press Series on Discrete Mathematics and its Applications, CRC Press, Boca Raton, 1998, Zbl 0943.11001.
5. R.A. Mollin, *Polynomials of Pellian type and continued fractions*, Serdica Math. J. **27** (2001), 317–342, MR1899042 (2003c:11139). MR1899042 (2003c:11139)
6. R.A. Mollin, *A continued fraction approach to the Diophantine equation* $ax^2 - by^2 = \pm 1$, JP Journal Algebra, Number Theory, and Appl. **4** (2004), 159–207, MR2049695 (2005a:11031), Zbl 1056.11017. MR2049695 (2005a:11031)
7. R.A. Mollin, *Lagrange, central norms, and quadratic Diophantine equations*, International J. Math. and Math. Sci. **7** (2005), 1039–1047. MR2170503
8. R.A. Mollin, *Necessary and sufficient conditions for the central norm to equal a power of* 2 *in the simple continued fraction expansion of* $\sqrt{D}$ *for any nonsquare* D¿1, Canadian Math. Bulletin **48** (2005), 121–132, MR2118769 (2005i:11013). MR2118769 (2005i:11013)
9. A.J. Stephens and H.C. Williams, *Some computational results on a problem of Eisenstein*, Théorie des nombres (Quebec, PQ, 1987) (J-M De Koninck and C. Levesque, eds.), Walter de Gruyter, Berlin, New York (1989), 869–886, MR1024611 (91c:11066), Zbl 0689.10024. MR1024611 (91c:11066)
10. H.C. Williams, *Eisenstein's problem and continued fractions*, Utilitas Math. **37** (1990), 145–158, MR1068514 (91h:11018), Zbl 0718.11010. MR1068514 (91h:11018)