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Lagrange, central norms, and quadratic Diophantine equations.

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Let D be a positive integer that is not a perfect square. The partial quotients in the continued fraction expansion of \sqrt{D} have the shape $q_0; q_1, \dots, q_{l-1}, 2q_0, \dots$, where \dots means that the sequence $q_1, \dots, q_{l-1}, 2q_0$ is repeated; this is the period of \sqrt{D} and l is its length. The complete quotients (not to be confused with the partial quotients of the continued fraction expansion) are given by $\frac{P_k + \sqrt{D}}{Q_k}$, where $P_0 = 0$, $Q_0 = 1$ and for $k \geq 1$, $P_{k+1} = q_k Q_k - P_k$, $D = P_{k+1}^2 + Q_k Q_{k+1}$. It is true that the integer part of the k -th complete quotient is equal to the k -th partial quotient. If l is even, $Q_{l/2}$ is called the central norm. Let (x_0, y_0) be the (positive) fundamental solution of the Pell equation $x^2 - Dy^2 = 1$. The following result, which generalizes a theorem of Lagrange, is proved. If $D > 2$ is not a perfect square then the following three conditions are equivalent: (i) $x_0 \equiv 1 \pmod{D}$. (ii) The equation $x^2 - Dy^2 = 2$ is solvable in integers. (iii) $l \equiv 0 \pmod{4}$ and the central norm is 2.

Among the other results of the paper, the most elegant is the following: If D is a positive integer which is not a perfect square such that the length l is even, then $x_0 \equiv \pm 1 \pmod{D}$ iff $Q_{l/2} = 2$.

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