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Algebras with uniformly distributed invariants.

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Let K be a finite abelian extension of the field of rational numbers $\mathbf{Q}$, and let L be the smallest cyclotomic field extension of $\mathbf{Q}$ which contains K. If A is a central simplealgebra over K, let $[A]$ denote its class in the Brauer group $B(K)$ of K. The Schur subgroup $S(K)$ of $B(K)$ consists of those classes that contain a simple component of a finite group algebra over K. Let p be any rational prime and let $\mathcal{P}$ be any prime of K lying over p. If $K_{\mathcal{P}}$ is the completion of K with respect to $\mathcal{P}$, the Brauer group of $K_{\mathcal{P}}$ is isomorphic to the additive group $\mathbf{Q}/\mathbf{Z}$ of rational numbers modulo 1. Consequently there is a homomorphism of $B(K)$ into $\mathbf{Q}/\mathbf{Z}$ which assigns to $[A]$ an element $\mathrm{inv}_{\mathcal{P}}(A)$ in $\mathbf{Q}/\mathbf{Z}$. M. Benard and M. M. Schacher [same J. **22** (1972), 378–385; MR0302747 (46 #1890)] have shown that the following are true for an element $[A]$ of $S(K)$: (1) If the index of A is m, then K contains a primitive mth root of unity ε_m. (2) If σ is an automorphism of K and b is an integer such that $\sigma(\varepsilon_m) = {\varepsilon_m}^b$, then $\mathrm{inv}_{\mathcal{P}}(A) = b\,\mathrm{inv}_{\sigma\mathcal{P}}(A)$. A central simple algebra A over K satisfying (1) and (2) is said to have uniformly distributed invariants, and the set of classes of these algebras forms a subgroup $U(K)$ of $B(K)$. After observing some pleasant properties of the group $U(K)$, the author establishes the following relations between $S(K)$ and $U(K)$. Let q be an odd prime and let $\varepsilon_q a$ be the highest q-power root of unity in K. If q does not divide $[K\colon Q(\varepsilon_q a)]$ then the q-primary parts of $S(K)$ and $U(K)$ are equal. But if $a > 0$, q divides $[K\colon Q(\varepsilon_q a)]$, and q does not divide $[L\colon K]$, then the q-primary part of $S(K)$ is a subgroup of infinite index in the q-primary part of $U(K)$. *H. F. Kreimer*