Advanced Number Theory with Applications
Richard A. Mollin

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MAA Review
[Reviewed by Michael Berg, on 10/15/2009]
This terrific book is testimony to Richard Mollin's mathematical erudition, wonderful taste, and also his breadth of culture. Mollin presents this work as engendering a second course in number theory (viz. his Fundamental Number Theory with Applications), and it certainly is that — and a lot more. In fact, the book has to be seen to believed: Mollin takes us from hard core (if basic) algebraic number theory and Dedekind ideal theory to quadratic forms, then to Diophantine approximations and arithmetic functions (and Riemann's zeta function). Then a quantum jump of sorts occurs: p-adic analysis (I'm reminded of Borevich-Shafarevich, of course). And after this we get a sequence of truly impressive crescendos: Dirichlet on primes in arithmetic progressions, Diophantine equations (including a fantastic discussion of the ABC conjecture and its famous implications (see pp. 294–300), elliptic curves (see below) and modular forms (see below, further down); and then there's an appendix on sieve methods. An amazing sweep!

Advanced Number Theory with Applications is also a lot of fun in an extra-mathematical sense: the book is riddled with exceptional biographical sketches (compact, yes, but also very evocative: see e.g. the sketch of Charles Hermite's life on p. 126, that of Cauchy on p. 239; then there's Jacob Bernoulli on p. 207 and Hermann Weyl on p. 31), and every section features an introductory quote by a luminary who might very well not be a mathematician. Here I simply must quote three favorites: "It remains an old maxim of mine that when you have excluded the impossible, whatever remains, however improbable must be the truth …" (Sherlock Holmes; cf. p. 55); "My mind rebels at stagnation. Give me a problem, give me my work, give me the most abstrusive cryptogram, or the most intricate analysis, and I am in my proper atmosphere …" (Holmes again; cf. p. 129: a bonanza for some one like me, a diehard fan of the great detective!); and then there is the following contemporary offering by UC Riverside's John de Pillis:

I can't cut this steak, he confided
To the waiter who simply recited,
Your prime cut of course
Is tough as a horse
Since you can't take a prime and divide it

(Painful, no?). Manifestly Mollin's book is eminently browsable.

But it is the mathematics that matters, of course, and in this connection I want to highlight two very telling illustrations, already alluded to above. Mollin's treatment of elliptic curves (pp. 301–331) is a model of clear exposition, pitched at an appropriate introductory level given the aforementioned audience, covering all the appropriate bases (modulo the more titanic or austere proofs, for which Mollin provides references), singling out the accessible and currently sexy topics of cryptography and primality testing, and sprinkling fine exercises throughout. The student following this route should emerge with a bird's eye view of a deep and viable part of contemporary mathematics and be poised to start exploring topics of choice in greater detail almost immediately. Indeed, Advanced Number Theory with Applications succeeds very well in its goal of providing a means of transition from more or less foundational material, to papers and advanced monographs, i.e. research in the field.

The second illustration I want to mention along these lines is the book's last chapter, "Modular forms," whose four sections comprise a Leitfaden to reckon with: the modular group, modular forms and functions, applications to elliptic curves, Shimura-Taniyama-Weil and Fermat's Last Theorem. [!] This is amazing stuff: good reduction on p. 357, conductors on p. 360, STW and Weil curves (resp. L-functions) on p. 363 (resp. 364), Ribet on p. 365, and then, in eight lines, the proof of FLT, also on p. 365. Granted, there is a lot of very hard mathematics swept under the rug, but what Mollin presents to his (novice) reader is deep, evocative, and very, very tantalizing.

Thus, Advanced Number Theory with Applications is a wondrous book, really, successfully fulfilling the author's purpose of effecting a bridge to modern number theory for the, shall we say, somewhat initiated. And for folks like me, now the front-runner for the honor of being my department's curmudgeon, replete with the age-requirement met, the book under review is immensely attractive for other reasons: I learned these things from a half dozen books or more, with some articles thrown in (and a few gaps remain): it's very nice to find in Mollin's book a high quality and coherent treatment of this beautiful material, and pointers in abundance to where to go next.

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