

AMAT 503 - Project
Due: Friday, April 13, 2007

Complete one of the following projects:

1. Write MATLAB code to compute the canonical dual window γ for a given Gabor frame

$$\mathcal{G}(g, \alpha, \beta) = \{T_{\alpha k} M_{\beta n} : k, n \in \mathbb{Z}\},$$

where g is any inputted window function. Test your code on some reasonable windows g and various values of α, β . Plot the results.

(Note: you may assume g has compact support. You may try to compute the dual window by finding the operator S^{-1} explicitly, or by solving the linear system $S\gamma = g$ for the unknown γ . Or, if you wish, read about the Zuck transform to find another method to solve for γ .)

2. Use the Fourier, STFT, Gabor, or wavelet transform to implement some concrete applications. For instance

- a pitch detector for musical instruments;
- noise reduction on a given signal (by reducing to zero the small wavelet coefficients, say);
- signal detection (eg. find a spike embedded in white Gaussian noise);
- note identification (eg. input a recording of music, output a chart of notes that were played)
- model of stock market valuations;
- nonstationary filtering of 2D images (eg. deblurring of a photo);
- your own ideas.

You may use some pre-packaged code to implement the transforms (eg MATLAB wavelet toolbox) but you must write your own code for the particular application. Test your code and give some meaningful output and plots that demonstrates that it works.

3. Make a technical presentation (either written, or presented in class) on some theoretical aspects of harmonic analysis. For instance:

- something along the lines of Ryan's presentation on locally compact groups;
- Fourier transforms on compact, nonabelian groups, including the Plancherel theorem;
- Fourier transforms on finite, nonabelian groups, including the Plancherel theorem;
- the Zuck transform and its application to Gabor systems;
- the Balian-Low theorem;
- constructing Daubechies wavelets;
- diagonalizing the Fourier transform, finding its eigenvalues and eigenfunctions;
- other suggestions.

4. Discuss with me any of your own ideas for a significant project, involving either numerical work or some theoretical aspect of Fourier and wavelet analysis.