

Find dy/dx for the following curves:

$$x^2y + e^x = \cos(y) \quad (x^3 + y^3)^5 + x^2 = ye^x \quad \sin(x^2y) + y^2 = x^3 - 2e^y$$

$$\frac{dy}{dx} = \frac{-e^x - 2xy}{x^2 + \sin(y)} \quad \frac{dy}{dx} = \frac{ye^x - 2x - 5(x^2 + y^3)^4(3x^2)}{5(x^3 + y^3)^4 3y^2 - e^x}$$

$$\frac{dy}{dx} = \frac{3x^2 - \cos(x^2y)2xy}{2e^y + 2y + \cos(x^2y)x^2}$$

Find the equation of the tangent line to the given curve at the specified point:

$$x^2 + xy + y^2 = 3 \text{ at } (1, 1)$$

$$\left. \frac{dy}{dx} \right|_{x=y=1} = \left. \frac{-2x - y}{x + 2y} \right|_{x=y=1} = \frac{-2 - 1}{1 + 2} = -1 \quad y - 1 = -(x - 1)$$

$$x^2 + y^2 = (2x + 2y^2 - x^2)^2 \text{ at } (0, 1/2)$$

$$\left. \frac{dy}{dx} \right|_{x=0, y=1/2} = \left. \frac{-2x + 2(2x + 2y^2 - x^2)(2 - 2x)}{2y - 2(2x + 2y^2 - x^2)4y} \right|_{x=0, y=1/2} = \frac{2}{1 - 2(1/2)2} = -2$$

$$y - \frac{1}{2} = -2(x - 0)$$

Use logarithmic differentiation to find the derivatives of the following functions:

$$f(x) = x^{\tan x} \quad g(x) = \frac{\sqrt{2x^3 + x^{-1/2}} (2x^3 + 5x)^2}{3x^8 + 17x^2} \quad h(x) = (3x^2 + 2 \sin x)^{x^2+1}$$

$$f'(x) = x^{\tan x} \left(\sec^2(x) \ln(x) + \frac{\tan x}{x} \right)$$

$$g'(x) = \frac{\sqrt{2x^3 + x^{-1/2}} (2x^3 + 5x)^2}{3x^8 + 17x^2} \left(\left(\frac{1}{2} \right) \frac{6x^2 - \frac{1}{2x^{3/2}}}{2x^3 + x^{-1/2}} + (2) \frac{6x^2 + 5}{2x^3 + 5x} - \frac{24x^7 + 34x}{3x^8 + 17x^2} \right)$$

$$h'(x) = (3x^2 + 2 \sin x)^{x^2+1} \left(2x \ln(3x^2 + 2 \sin x) + \frac{(x^2 + 1)(6x + 2 \cos x)}{3x^2 + 2 \sin x} \right)$$

Find a linearization to the following functions at the specified point:

$$f(x) = \sin(2x) \text{ at } x = 0 \quad L(x) = 2x$$

$$f(x) = \sqrt{1+x} \text{ at } x = 8 \quad L(x) = 3 + \frac{1}{6}(x - 8)$$

- Use a linear approximation to find a good estimate for $\sqrt{15}$.

$$L(x) = 4 + \frac{1}{8}(x - 16), \quad L(15) = 4 - \frac{1}{8} = \frac{31}{8}$$

Evaluate the following limits:

$$\lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{x - 9} = \frac{1}{6}$$

$$\lim_{x \rightarrow 1} \frac{x^3 + 2x - 3}{x - 1} = 5$$

$$\lim_{x \rightarrow 0} \frac{\sin 3x}{2x} = \frac{3}{2}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2} = 2$$

$$\lim_{x \rightarrow \infty} \sqrt{x^2 + x} - x = \frac{1}{2}$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x^2}\right)^x = 1$$