Math 353 Quiz 5

Tuesday, April 12

NAME:

JUSTIFY YOUR ANSWERS. Answer each question in the space provided. A correct answer without work shown may be worth 0 points, while an incorrect answer with full justifiction may be worth partial credit. Non-graphing calculators are allowed. Each question is worth 5 points.

1. Evaluate the surface integral $\int\int_S 8y\ dS$ where S is the graph of the function $z = 4 - y^2$ with $0 \le x \le 2$ and $z \ge 0$.

   The normal vector is given by $n = \langle -\nabla z, 1\rangle = \langle 0, 2y, 1\rangle$. Its length is the surface area element, $|n| = \sqrt{4y^2+1}$. When $z \ge 0$, we have $-2 \le y \le 2$. We are given $0 \le x \le 2$. Thus

$$\int\int_S 8y\ dS = \int_{-2}^{2}\int_0^2 8y\sqrt{4y^2+1}\ dx\ dy = 0$$

2. Find the surface area of the part of the cylinder $x^2 + y^2 = 4$ with $x \ge 0$ between $z = 0$ and the plane $z = (1/2)x$.

   This is a vertical surface, so we must parametrize it. One good parametrization of the surface is

$$r(u, v) = \{2\cos(u), 2\sin(u), v\}$$

   on the domain $D = \{(u, v) \mid -\pi/2 \le u \le \pi/2, 0 \le v \le \cos(u)\}$ (this comes from $x \ge 0$ and $0 \le z \le 1/2\ x$). Then $r_u(u, v) = \langle -2\sin(u), 2\cos(u), 0\rangle$ and $r_v(u, v) = \langle 0, 0, 1\rangle$. The cross product is $r_u \times r_v = \langle 2\cos(u), -2\sin(u), 0\rangle$ and its length is $|r_u \times r_v| = 2$. This is the surface area element. Thus the surface area is

$$\int\int_S 1\ dS = \int_{-\pi/2}^{\pi/2}\int_0^{\cos(u)} 2\ dv\ du = \int_{-\pi/2}^{\pi/2} 2\cos(u)\ du = 4$$

3. Find the flux $\int\int_S F \cdot dS$ out of the surface S, where $F(x, y, z) = \langle x, y, z\rangle$ and S is the union of $z = \sqrt{4 - x^2 - y^2}$ and its bottom $z = 0$.

   Let S_1 be the surface which is the part of the sphere and S_2 be the flat piece on the bottom. We'll compute the flux out of S_1 first. In this case the surface is given as the graph of a function whose domain is the disk $\{x^2 + y^2 \le 4\}$. You can compute the normal vector right away:

$$n = \pm\langle -\nabla, 1\rangle = \pm\langle \frac{x}{\sqrt{4-x^2-y^2}}, \frac{y}{\sqrt{4-x^2-y^2}}, 1\rangle$$

Since we want the flux OUT of the surface, we want the normal vector to point UP, so we choose $n = \langle \frac{x}{\sqrt{4-x^2-y^2}}, \frac{y}{\sqrt{4-x^2-y^2}}, 1\rangle$. Then the flux is

$$\int\int_{S_1} F{\cdot}dS = \int\int_{x^2+y^2\leq 4} \langle x, y, z\rangle{\cdot}\langle \frac{x}{\sqrt{4-x^2-y^2}}, \frac{y}{\sqrt{4-x^2-y^2}}, 1\rangle \; dx \; dy = \int\int_{x^2+y^2\leq 4} \frac{4}{\sqrt{4-x^2-y^2}} dxdy$$

$$= \int_0^{2\pi}\int_0^2 \frac{4r}{\sqrt{4-r^2}} \; dr \; d\theta = 16\pi$$

We still have to compute the flux out of S_2. Since this is flat, the unit normal vector is $\langle 0, 0, -1\rangle$. It is pointing DOWN since we want the flux OUT of S. Then

$$\int\int_{S_2} F \; dS = \int\int_{x^2+y^2\leq 4} \langle x, y, z\rangle \cdot \langle 0, 0, -1\rangle dxdy = \int\int -zdxdy = 0$$

since $z = 0$.

The total flux is thus 16π.