Math 353 Quiz 5

Tuesday, April 12

NAME:

JUSTIFY YOUR ANSWERS. Answer each question in the space provided. A correct answer without work shown may be worth 0 points, while an incorrect answer with full justifiction may be worth partial credit. Non-graphing calculators are allowed. Each question is worth 5 points.

1. Find the surface area of the part of the cone $z = \sqrt{x^2 + y^2}$ which is in the first octant and lies inside the cylinder $x^2 + y^2 = 9$.

   In this case, the surface is given as the graph of a function, so its normal vector is easy to compute:

$$n = \pm\langle -\nabla, 1\rangle = \langle -\frac{x}{\sqrt{x^2+y^2}}, -\frac{y}{\sqrt{x^2+y^2}}, 1\rangle$$

   Its length is the surface area element. It is:

$$|n| = \sqrt{\frac{x^2}{x^2+y^2} + \frac{y^2}{x^2+y^2} + 1} = \sqrt{2}$$

   The domain of the function is $D = \{x^2 + y^2 \leq 9, x \geq 0, y \geq 0\}$. Thus the surface area is

$$\int\int_S 1 \; dS = \int\int_D \sqrt{2} \; dx \; dy = \int_0^{\pi/2} \int_0^3 \sqrt{2} r \; dr \; d\theta = 9\sqrt{2}\pi/4$$

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2. Evaluate the surface integral $\int\int_S y \; dS$ where S is the part of the cylinder $x^2 + y^2 = 4$ with $x \geq 0$ between $z = 0$ and the plane $z = (1/2)x$.

   In this case the surface is vertical, so we have to parametrize it. One good parametrization of the surface is

$$r(u, v) = \{2\cos(u), 2\sin(u), v\}$$

   on the domain $D = \{(u, v) \mid -\pi/2 \leq u \leq \pi/2, 0 \leq v \leq \cos(u)\}$ (this comes from $x \geq 0$ and $0 \leq z \leq 1/2 \; x$). Then $r_u(u, v) = \langle -2\sin(u), 2\cos(u), 0\rangle$ and $r_v(u, v) = \langle 0, 0, 1\rangle$. The cross product is $r_u \times r_v = \langle 2\cos(u), -2\sin(u), 0\rangle$ and its length is $|r_u \times r_v| = 2$. This is the surface area element. Thus the surface integral is given by

$$\int\int_S y \; dS = \int_{-\pi/2}^{\pi/2} \int_0^{\cos(u)} 4\sin(u) \; dv \; du = 2\sin^2(u)|_{-\pi/2}^{\pi/2} = 0$$

3. Find the flux $\int\int_S F \cdot dS$ out of the surface S, where $F(x, y, z) = \langle xz, yz, z^2\rangle$ and S is the union of $z = x^2 + y^2$, $0 \leq z \leq 4$ and its top $z = 4$.

Let S_1 be the surface which is part of the paraboloid, and S_2 be the flat part of the surface where $z = 4$. We'll compute the flux out of S_1 first. In this case the surface is given as the graph of a function whose domain is the disk $\{x^2 + y^2 \leq 4\}$. You can compute the normal vector right away:

$$n = \pm\langle -\nabla, 1\rangle = \pm\langle -2x, -2y, 1\rangle$$

Since we want the flux OUT of S, we want the normal vector to point DOWN, so we choose $n = \langle 2x, 2y, -1\rangle$. Then the flux is

$$\int\int_{S_1} F \cdot dS = \int\int_{x^2+y^2\leq 4} \langle xz, yz, z^2\rangle \cdot \langle -2x, -2y, 1\rangle dxdy = \int\int_{x^2+y^2\leq 4} -2x^2z - 2y^2z + z^2 dxdy$$

No remember that $z = x^2 + y^2$ to get

$$\int\int_{x^2+y^2\leq 4} -(x^2 + y^2)^2 dxdy = \int_0^{2\pi}\int_0^2 r^5 dr d\theta = 64\pi/3$$

We still have to compute the flux out of S_2. Since this is flat, the unit normal vector is $\langle 0, 0, 1\rangle$. It is pointing UP since we want the flux OUT of S. Then

$$\int\int_{S_2} F\ dS = \int\int_{x^2+y^2\leq 4} \langle xz, yz, z^2\rangle \cdot \langle 0, 0, 1\rangle dxdy = \int\int_{x^2+y^2\leq 4} z^2 dxdy$$

But $z = 4$, so this is just

$$\int\int_{x^2+y^2\leq 4} 16 dxdy = 16(\text{area of the disk}) = 64\pi$$

Thus the total flus is $64\pi/3 + 64\pi$.