Math 353 Quiz 4

Tuesday, March 29

NAME:

JUSTIFY YOUR ANSWERS. Answer each question in the space provided. A correct answer without work shown may be worth 0 points, while an incorrect answer with full justifiction may be worth partial credit. Non-graphing calculators are allowed. Each question is worth 5 points.

1. Find a parametrization oof the curve C of intersection of the paraboloid $z = (\frac{x}{3})^2 + y^2$ and the ellipsoid $x^2 + 9y^2 + z^2 = 10$.

   Rewrite the first equation as $9z = x^2 + 9y^2$ and substitute it into the second equation to get

   $$z^2 + 9z - 10 = 0$$

   This has solutions $z = -10$ and $z = 1$. Since $z = (x/3)^2 + y^2 \geq 0$, we can discard the solution $z = -10$. Substitution $z = 1$ into either equation, we get $x^2 + 9y^2 = 9$, which is the equation of an ellipse centered at the origin. The parametrization is thus

   $$r(t) = \langle 3\cos(t), \sin(t), 1\rangle$$

   with $0 \leq t \leq 2\pi$.

2. Evaluate the integral $\int_C f\ ds$ where $f(x, y, z) = \frac{16}{3}xy$ and C is the curve of intersection from problem (1).

   From problem (1), we have $r(t) = \langle 3\cos(t), \sin(t), 1\rangle$. Then $r'(t) = \langle -3\sin(t), \cos(t), 0$ and $|r'(t)| = \sqrt{9\cos^2(t) + \sin^2(t)}$. We have $f(r(t)) = \frac{16}{3}3\cos(t)\sin(t) = 16\cos(t)\sin(t)$ so that

   $$\int_C f\ ds = \int_0^{2\pi} 16\cos(t)\sin(t)\sqrt{9\cos^2(t) + \sin^2(t)}\ dt$$

   Use $u = 9\cos^2(t) + \sin^2(t)$ and substitute to evaluate this integral. You get:

   $$-(2/3)(9\cos^2(t) + \sin^2(t))|_0^{2\pi} = 0$$