Math 353 Quiz 4

Thursday, March 24

NAME:

JUSTIFY YOUR ANSWERS. Answer each question in the space provided. A correct answer without work shown may be worth 0 points, while an incorrect answer with full justifiction may be worth partial credit. Non-graphing calculators are allowed. Each question is worth 5 points.

1. Find a parametrization oof the curve C of intersection of the paraboloid $z = \frac{1}{3}(x^2+y^2)$ and the sphere $x^2+y^2+z^2=4$.

   Rewrite the first equation as $3z = x^2+y^2$ and substitute it into the second equation to get

   $$z^2+3z-4=0$$

   This has solutions $z=-4$ and $z=1$. Since $z=(1/3)(x^2+y^2)\geq 0$, we can discard the solution $z=-4$. Substitution $z=1$ into either equation, we get $x^2+y^2=3$, which is the equation of a circle of radius $\sqrt{3}$ centered at the origin. The parametrization is thus

   $$r(t) = \langle \sqrt{3}\cos(t), \sqrt{3}\sin(t), 1\rangle$$

   with $0 \leq t \leq 2\pi$.

2. Evaluate the integral $\int_C f\ ds$ where $f(x,y,z) = z-x$ and C is the curve of intersection from problem (1).

   From problem (1), we have $r(t) = \langle \sqrt{3}\cos(t), \sqrt{3}\sin(t), 1\rangle$. Then $r'(t) = \langle -\sqrt{3}\sin(t), \sqrt{3}\cos(t), 0$ and $|r'(t)| = \sqrt{3\cos^2(t)+3\sin^2(t)} = \sqrt{3}$. We have $f(r(t)) = 1-\sqrt{3}\cos(t)$, so that

   $$\int_C f\ ds = \int_0^{2\pi} (1-\sqrt{3}\cos(t))(\sqrt{3})\ dt = \sqrt{3}(t-\sqrt{3}\sin(t))|_0^{2\pi} = 2\pi$$

3. Evaluate $\int_C F\ ds$ where $F(x,y,z) = \langle \sin(x), \cos(y), xz\rangle$ and C is defined parametrically by $\vec{r}(t) = \langle t^3, -t^2, 1\rangle$, $0\leq t\leq 1$.

   We have $\vec{r}'(t) = \langle 3t^2, -2t, 0\rangle$ and $F(\vec{r}(t)) = \langle \sin(t^3), \cos(-t^2), t^3\rangle$. Thus

   $$\int_C F\ ds = \int_0^1 F(\vec{r}(t))\cdot\vec{r}'(t)\ dt = \int_0^1 3t^2\sin(t^3) - 2t\cos(-t^2)\ dt$$

   $$= -\cos(t^3) + \sin(-t^2)|_0^1 = 1 - \sin(-1) - \cos(1)$$