Math 353 Quiz 3

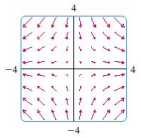
Tuesday, March 15

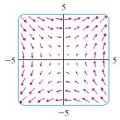
NAME:

JUSTIFY YOUR ANSWERS. Answer each question in the space provided. A correct answer without work shown may be worth 0 points, while an incorrect answer with full justifiction may be worth partial credit. Non-graphing calculators are allowed. Each question is worth 5 points.

1. Which of the following sketches could be the vector field of $F = \langle 2x, -2y \rangle$.

   It's the first one. Look at points where $y = 0$. Since $F(x, 0) = \langle 2x, 0 \rangle$ the vectors at these points should be parallel to the x-axis. Only the first vector field has this property.





2. Either find a potential function for the vector field $F(x, y) = \langle y^2, 2xy + 1 \rangle$ or else prove that the vector field is not conservative.

   A quick check using the mixed partial derivative test shows that the vector field COULD be conservative. If $\frac{\partial}{\partial x}\phi(x, y) = y^2$, then $\phi(x, y) = xy^2 + f(y)$. Then

$$\frac{\partial}{\partial y}\phi(x, y) = 2xy + f'(y) = 2xy + 1$$

   which implies that $f'(y) = 1$, so $f(y) = y + C$. Any of the functions

$$\phi(x, y) = xy^2 + y + C$$

   will be a potential function for this vector field.