Math 353 Quiz 2

Tuesday, February 8

NAME:

JUSTIFY YOUR ANSWERS. Answer each question in the space provided. A correct answer without work shown may be worth 0 points, while an incorrect answer with full justifiction may be worth partial credit. Non-graphing calculators are allowed. Each question is worth 5 points.

1. Find the absolute maximum and absolute minimum of $f(x, y) = x^2 - y^3$ on the disk $D = \{(x, y)|x^2 + y^2 \leq 2\}$.

   First we find the interior critical points. The gradient is $\nabla f(x, y) = \langle 2x, -3y^2 \rangle$. This is equal to $\langle 0, 0 \rangle$ when $(x, y) = (0, 0)$. We have $f(0, 0) = 0$.

   Now we look for the extreme values on the boundary. The constrain equaation is $g(x, y) = x^2 + y^2 = 2$. The gradient is $\nabla g(x, y) = \langle 2x, 2y \rangle$. We obtain three equations:

$$2x = 2\lambda x$$

$$3y^2 = 2\lambda y$$

$$x^2 + y^2 = 2$$

   From the first equationl, either $x = 0$ or $\lambda = 1$. From the second equation either $y = (2/3)\lambda$ or $y = 0$. Thus, if $x \neq 0$ and $y \neq 0$ then $y = (2/3)(1) = 2/3$. The third equation takes care of the rest: if $x = 0$ then $y = \pm\sqrt{2}$; if $y = 0$ then $x = \pm\sqrt{2}$; if $y = 2/3$ then $x = 14/9$. The corresponding values are:

$$f(\pm\sqrt{2}, 0) = 2; \qquad f(0, \sqrt{2}) = -2\sqrt{2}; \qquad f(0, -\sqrt{2}) = 2\sqrt{2}$$

$$f(14/9, 2/3) = 196/81 - 8/27 = 172/27 \approx 6.4$$

   Thus the maximum occurs at $(14/9, 2/3)$ and the minimum occurs at $-2\sqrt{2}$.

2. Evaluate the iterated integral $\int_0^{\pi/2}\int_0^{\sin(x)} \frac{x}{\sin(x)} dydx$.

   The order of integration is crucial here. We have

$$\int_0^{\sin(x)} \frac{x}{\sin(x)} dy = \frac{x}{\sin(x)} y|_0^{\sin(x)} = x$$

   And then

$$\int_0^{\pi/2} xdx = (1/2)x^2|_0^{\pi/2} = \pi^2/8$$

3. For the integral $\int_0^{\pi/2}\int_0^{\sin(x)} \frac{x}{\sin(x)} dydx$

   (a) Describe the region D over which the integral is taken.

   (b) Switch the order of integration for the integral above (but do not evaluate).

   (a) $D = \{(x, y)|0 \leq x \leq \pi/2;\ 0 \leq y \leq \sin(x)\} = \{(x, y)|0 \leq x \leq \arcsin(y);\ 0 \leq y \leq 1\}$

   (b) $\int_0^1 \int_0^{\arcsin(y)} \frac{x}{\sin(x)} dxdy$, which is difficult to integrate.