Math 353 Quiz 2

Thursday, February 10

NAME:

JUSTIFY YOUR ANSWERS. Answer each question in the space provided. A correct answer without work shown may be worth 0 points, while an incorrect answer with full justifiction may be worth partial credit. Non-graphing calculators are allowed. Each question is worth 5 points.

1. Find the absolute maximum and absolute minimum of $f(x, y) = x^3 - y^2$ on the region $D = \{(x, y)|x^2 + 3y^2 \leq 9\}$.

   Ok, first we'll find the interior critical points. The gradient is $\nabla f(x, y) = \langle 3x^2, -2y\rangle$. This is equal to $\langle 0, 0\rangle$ when $(x, y) = (0, 0)$. This is the only interior critical point, and we have $f(0, 0) = 0$.

   Now we have to find the maximum and minimum values on the boundary. The constraint equation is $g(x, y) = x^2 + 3y^2 = 9$. It's gradient is $\nabla g(x, y) = \langle 2x, 6y\rangle$. We obtain three equations

   $$3x^2 = 2\lambda x$$

   $$-2y = 6\lambda y$$

   $$x^2 + 3y^2 = 9$$

   From the first equation, either $x = 0$ or $x = (2/3)\lambda$. From the second equation, either $y = 0$ or $\lambda = -1/3$. Thus, if neither x nor y is 0, then $x = (2/3)(-1/3) = -2/9$. The third equation takes care of the rest: if $x = 0$ then $y = \pm\sqrt{3}$; if $y = 0$ then $x = \pm 3$; if $x = -2/9$ then $y = 725/243 \approx 3$. We check these potential extreme values:

   $$f(0, \pm\sqrt{3}) = -3; \qquad f(3, 0) = 27; \qquad f(-3, 0) = -27$$

   $$f(-2/9, 725/243) \approx -8/729 - 9 \approx -9$$

   Thus the absolute maximum occurs at $(3, 0)$ and the absolute minimum occurs at $(-3, 0)$.

2. Evaluate the iterated integral $\int_0^1 \int_0^{\arctan(x)} \frac{\sqrt{x}}{\arctan(x)} dydx$.

The order of integration is crucial in this problem.

$$\int_0^1 \int_0^{\arctan(x)} \frac{\sqrt{x}}{\arctan(x)} dydx = \int_0^1 \frac{\sqrt{x}}{\arctan(x)} y|_0^{\arctan(x)} dx = \int_0^1 \sqrt{x} dx$$

The rest is a simple single variable integral, just remember that $\sqrt{x} = x^{1/2}$.

$$\int_0^1 \sqrt{x} dx = (2/3)x^{3/2}|_0^1 = 2/3$$

3. For the integral $\int_0^1 \int_0^{\arctan(x)} \frac{\sqrt{x}}{\arctan(x)} dydx$

(a) Describe the region D over which the integral is taken.

(b) Switch the order of integration for the integral above (but do not evaluate).

(a) $D = \{(x, y)|0 \leq y \leq \arctan(x); 0 \leq x \leq 1\} = \{(x, y) : 0 \leq y \leq \pi/4; 0 \leq x \leq \tan(y)\}$.

(b) $\int_0^{\pi/4} \int_0^{\tan(y)} \frac{\sqrt{x}}{\arctan(x)} \, dxdy$, which is impossible to integrate.