Math 353 Quiz 1

Thursday, January 27

NAME:

JUSTIFY YOUR ANSWERS. Answer each question in the space provided. A correct answer without work shown may be worth 0 points, while an incorrect answer with full justifiction may be worth partial credit. Each question is worth 5 points.

1. Let $S = \{(x, y)|0 < |x| \le 1; 0 < |y| \le 1\}$.

   (a) Find the boundary δS.

   (b) Is S open? closed? neither?

   The boundary is the set

   $\delta S = \{(x, y) : x = \pm 1, -1 \le y \le 1\} \cup \{(x, y) : y = \pm 1, -1 \le x \le 1\} \cup \{(x, y)|x = 0, -1 \le y \le 1\} \cup \{(x, y) : y = 0, -1 \le x \le$

   Most people missed the axes, but did include the boundary of the square.

   The set S is neither open nor closed: those points on the outside of the square (i.e. $x = 1$ or $y = 1$ with $x \neq 0$ and $y \neq 0$) are contained in S. However, the points on the axes (i.e. $x = 0$ or $y = 0$) are not contained in S. Thus S contains some, but not all, of its boundary points.

2. Find and classify the critical points of the function $f(x, y) = e^{(x^2 - x^2 y + y^2)}$.

   The gradient is

   $$\nabla f(x, y) = \langle (2x - 2xy)e^{(x^2 - x^2 y + y^2)}, (-x^2 + 2y)e^{(x^2 - x^2 y + y^2)} \rangle$$

   There are no singular points. The critical point occur when the equations

   $$2x - 2xy = 2x(1 - y) = 0 \qquad - x^2 + 2y = 0$$

   are simultaneously satisfied. The first equation is satisfied when $x = 0$ or $y = 1$. When $x = 0$, the second equation indicates that $y = 0$ as well. When $y = 1$, the second equation indicates that $x = \pm\sqrt{2}$. So we have three critical point to classify: $(0, 0), (\sqrt{2}, 1), (-\sqrt{2}, 1)$.

   We'll classify these using the second derivative test. First, we calculate:

   $$f_{xx}(x, y) = (2x - 2xy)^2 e^{(x^2 - x^2 y + y^2)} + (2 - 2y)e^{(x^2 - x^2 y + y^2)}$$

   $$f_{yy}(x, y) = (-x^2 + 2y)e^{(x^2 - x^2 y + y^2)} + (2)e^{(x^2 - x^2 y + y^2)}$$

   $$f_{xy}(x, y) = (2x - 2xy)(-x^2 + 2y)e^{(x^2 - x^2 y + y^2)} + (-2x)e^{(x^2 - x^2 y + y^2)}$$

   Using the formula $D = f_{xx} f_{yy} - f_{xy}^2$ (some of you used the fomula in the book, which differs by a sign - as long as you use the book's rule for the 2-nd derivative test, that's ok) we have

   $$D(0, 0) = (2)(2) - 0 > 0 \qquad f_{xx}(0, 0) = 2 > 0$$

$$D(\sqrt{2}, 1) = -(-2\sqrt{2}e)^2 < 0$$
$$D(-\sqrt{2}, 1) = -(2\sqrt{2}e)^2 < 0$$

We can conclude that $(0, 0)$ is a local minimum, and $(\pm\sqrt{2}, 1)$ are saddle points.

3. Find and classify the singular points of the function $g(x, y) = \sqrt{x^4 + y^4}$.

   The gradient is
   $$\nabla g(x, y) = \langle \frac{2x^3}{\sqrt{x^4 + y^4}}, \frac{2y^3}{\sqrt{x^4 + y^4}} \rangle$$
   There are no critical points. The only singular point occurs at $(0, 0)$. Since $g(0, 0) = 0$, and since $g(x, y) \geq 0$ for all (x, y), the singular point must be an absolute minimum.

   Can you classify the singular point of $h(x, y) = \sqrt{x^3 + y^3}$?