ID:_____

1 Let $S = \{(\rho, \theta) | 0 \leq \theta < 2\pi, \rho = \theta\}$ be a set in the plane described in polar coordinates. The set S is not closed, but it can be made closed by adding a single point. Which point?

2 Let S be the three dimensional region which is ABOVE the plane $z = 0$, BELOW the hemisphere $x^2 + y^2 + z^2 = 4$ and OUTSIDE the cone $z = \sqrt{x^2 + y^2}$.

(a) Describe the region using spherical coordinates.

(b) Describe the region using cylindrical coordinates.

(c) Write the boundary ∂S in either spherical or cylindrical coordinates.

3 Find the points on the sphere $x^2 + y^2 + z^2 = 4$ that are closest and farthest from the point $(3, 1, -1)$.

4 Find the extreme values of the function $f(x, y) = -xy$ on the domain $x^2 + 4y^2 \leq 1$.

5 Find the extreme values of the function $f(x, y) = e^{-xy}$ on the domain $x^2 + 4y^2 \leq 1$.

6 Find the extreme values of the function $f(x, y) = cos(xy)$ on the triangular domain bounded by the x-axis, the y-axis, and the line $y = 1 - x$.

7 Evaluate the integral $\int\int_S \frac{1}{1+xy} dA$ where S is the wedge bounded by the x-axis and the line $x = y$.

8 Evaluate the integral $\int\int_S e^{x^2+y^2+1} dA$ where S is the region in the first quadrant bounded by the x-axis, the y-axis and the circle $x^2 + y^2 = 1$.

9 Convert the integral

$$\int_0^{2\pi} \int_0^{\sqrt{2}} \int_0^{\sqrt{4-r^2}} 3 \; dz \; r dr \; d\theta$$

to rectangular coordinates with order of integration $dz \; dx \; dy$, and to spherical coordinates.

END OF EXAM