Quiz 4
Math 349 Lecture 01

Thursday, November 18 2004

NAME:

JUSTIFY YOUR ANSWERS. Answer each question in the space provided. A correct answer without work shown may be worth 0 points, while an incorrect answer with full justifiction may be worth partial credit. Each question is worth 5 points.

1. Let $f(x, y) = \frac{x}{2x+y}$.

   (a) Find the domain of f. Describe precisely all values of x and y which are not in the domain.

   (b) Sketch the level curves of $f(x, y)$ with $c = -1, 0, 1, \ldots$. Make sure to label which level curve is which.

   (c) Describe the range.

   (a) The domain of f is all values of $\mathbb{R}^2$ except for those satisfying $2x + y = 0$. That is, f is defined everywhere except for the line $y = -2x$.

   (b) The level curves are the curves in the xy-plane which satisfy $c = \frac{x}{2x+y}$. When $c = 0$, the level curve is the line $x = 0$ minus the origin (since it's not in the domain of the function). When $c \neq 0$, the level curve is

   $$2cx - x + cy = 0.$$

   Divide by c, and solve for y:

   $$y = (1/c - 2)x.$$

   These are all lines through the origin. The level curves will be missing the point at the origin, since this is not in the domain of the function.

   (c) Since the level curves are defined for all values of c, the range is $\mathbb{R}$.

2. Calculate $\lim_{(x,y)\to(0,0)} \frac{x^3}{x^2+y^2}$ or say why it does not exist.

   We'll use the Squeeze theorem. An alternative solution is given by using the definition of the derivative. We want to show that the limit is equal to 0. Let's bound the absolute value:

   $$|\frac{x^3}{x^2+y^2}| = |x||\frac{x^2}{x^2+y^2}| \leq |x||\frac{x^2+y^2}{x^2+y^2}| = |x|$$

   That means that $-|x| \leq \frac{x^3}{x^2+y^2} \leq |x|$. By the Squeeze theorem,

   $$0 = \lim_{(x,y)\to(0,0)} -|x| \leq \lim_{(x,y)\to(0,0)} \frac{x^3}{x^2+y^2} \leq \lim_{(x,y)\to(0,0)} |x| = 0$$

   and we conclude that $\lim_{(x,y)\to(0,0)} \frac{x^3}{x^2+y^2} = 0$.

3. Find the partial derivatives of $f(x, y) = xe^{xy}$ at $x = -1$, $y = 0$.

$$\frac{\partial f}{\partial x} = xye^{xy} + e^{xy}$$

$$\frac{\partial f}{\partial y} = x^2 e^{xy}$$

Now plug in $x = -1$, $y = 0$:

$$f_x(-1, 0) = 1$$

$$f_y(-1, 0) = 1.$$