Quiz 3
Math 349 Lecture 01

October 14 2004

NAME:

JUSTIFY YOUR ANSWERS. Answer each question in the space provided. A correct answer without work shown may be worth 0 points, while an incorrect answer with full justifiction may be worth partial credit. Each question is worth 5 points.

1. Is the series

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n^2-1}}$$

absolutely convergent, conditionally convergent or divergent? Explain.

Solution:

Absolute Convergence: We use the comparison test to show that the series $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^-1}}$ diverges. If $n > 0$, we have that $0 < \sqrt{n^2-1} < \sqrt{n^2} = n$. Thus

$$\frac{1}{n} < \frac{1}{\sqrt{n^2-1}}$$

Since the series $\sum_{n=2}^{\infty} \frac{1}{n}$ diverges, so does $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^2-1}}$.

Conditional Convergence: Since the terms $\frac{1}{\sqrt{n^2-1}}$ are decreasing, the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n^2-1}}$ converges by the Alternating series test.

Thus the series is conditionally convergent, but not absolutely convergent.

2. What is the center, radius and interval of convergence for

$$\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n3^n}.$$

Solution: Rewrite the series as $\sum_{n=1}^{\infty} \frac{3^n(x-2/3)^n}{n3^n} = \sum_{n=1}^{\infty} \frac{1}{n}(x-2/3)^n$. Then it is easy to see that the center is $c = 2/3$. To find the radius of convergence R, compute

$$L = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{n}{n+1} = 1$$

where $a_n = 1/n$. Then $R = 1/L = 1$. Thus the interval of convergence is at least $(2/3-1, 2/3+1) = (-1/3, 5/3)$. We have to test the endpoints. When $x = -1/3$, the series becomes

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n}$$

which converges by the alternating series test (it is the alternating harmonic series). On the other hand, when $x = 5/3$ the series becomes

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

which is the harmonic series, and so diverges.

Thus the interval of convergence is $[-1/3, 5/3)$.

3. Find a Maclaurin series for $\ln \frac{1+x}{1-x}$ and find the radius of convergence.

   Write $\ln\left(\frac{1+x}{1-x}\right) = \ln(1+x) - \ln(1-x)$. Then use the known Maclaurin series:

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n$$

   which is valid on the interval of convergence $-1 < x \leq 1$. We'll need to find a series for $\ln(1-x)$. Let $t = -x$. Then

$$\ln(1-x) = \ln(1+t) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} t^n = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} (-x)^n = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}(-1)^n}{n} x^n = \sum_{n=1}^{\infty} \frac{-1}{n} x^n$$

   and the interval of convergence for this series is $-1 < t \leq 1$. That is, $-1 \leq x < 1$. (Note that for every n, either $(-1)^n = -1$ or $(-1)^{n-1} = -1$.)

   On the intersection of the intervals of convergence, we may add the two series together. That is,

$$\ln(1+x) - \ln(1-x) = \left(\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n\right) - \left(\sum_{n=1}^{\infty} \frac{-1}{n} x^n\right) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} + 1}{n} x^n$$

   and the interval of convergence is $-1 < x < 1$ (so the radius of convergence is 1). Note that some terms are 0. We could remedy this by rewriting this as

$$\sum_{n=1}^{\infty} \frac{2}{2n-1} x^{2n-1}.$$