Quiz 2
Math 349 Lecture 01

October 5

NAME:

JUSTIFY YOUR ANSWERS. Answer each question in the space provided. A correct answer without work shown may be worth 0 points, while an incorrect answer with full justifiction may be worth partial credit. Each question is worth 5 points.

1. Find the sum of the series:

$$\sum_{n=2}^{\infty} \frac{1}{n^2-1}$$

by using partial fractions.

This is a telescoping series. First we want to express the terms of the series as a sum of fractions. Write $\frac{1}{n^2-1} = \frac{1}{n-1}\frac{1}{n+1} = \frac{A}{n-1} + \frac{B}{n+1}$. Now we solve

$$1 = A(n+1) + B(n-1)$$

This must be true for all values of n. So we plug in $n = 2$ and $n = 3$ to get us started with two equations in two unknowns:

$$1 = 3A + B$$

$$1 = 4A + 2B$$

Subtract the first equation from the second to get $A = -B$. Use this to solve $A = -1/2$ and $B = 1/2$. Thus the telescoping series has n-th summand

$$s_n = (1/2-1/6)+(1/4-1/8)+(1/6-1/10)+\cdots+((1/2)\frac{1}{n-2}-1/2\frac{1}{n})+((1/2)\frac{1}{n-1}-(1/2)\frac{1}{n+1}) = 1/2+1/4-1/(2n)-1$$

To find the sum, take the limit as n tends to infinity: $\lim_{n\to\infty} s_n = \lim_{n\to\infty} 3/4-1/(2n)-1/[2(n+1)] = 3/4$.

2. Is the series

$$\sum_{n=1}^{\infty} ne^{-n^2}$$

convergent or divergent? Explain.

We use the Integral Test. First note that $ne^{-n^2} > 0$ when $n > 0$ and that the terms of the series are decreasing (since the associated function is decreasing).

Now, $\int xe^{-x^2}dx = (-1/2)e^{-x^2} + C$ by substitution. So,

$$\int_1^{\infty} xe^{-x^2}dx = \lim_{b\to\infty}[(-1/2)e^{-b^2} - (-1/2)e^{-1}] = (1/2)e^{-1}$$

Since the integral converges, the series converges as well.

3. Is the series

$$\sum_{n=1}^{\infty}\left(\frac{1+\sin(n)}{\sqrt{n}}\right)^3$$

convergent or divergent? Explain.

Since $-1 \le \sin n \le 1$, we have $0 \le 1+\sin n \le 2$. Thus $0 \le \left(\frac{1+\sin(n)}{\sqrt{n}}\right)^3 \le (2/\sqrt{n})^3 = 8n^{-3/2}$. The series $\sum_n = 1^{\infty} 8\left(\frac{1}{n}\right)^{3/2}$ converges as it is a p-series with $p > 1$. Thus the series $\sum_{n=1}^{\infty}\left(\frac{1+\sin(n)}{\sqrt{n}}\right)^3$ converges by comparison.