Quiz 2
Math 349 Lecture 01

September 30

NAME:

JUSTIFY YOUR ANSWERS. Answer each question in the space provided. A correct answer without work shown may be worth 0 points, while an incorrect answer with full justifiction may be worth partial credit. Each question is worth 5 points.

1. Find the sum of the series:
$$\sum_{n=3}^{\infty} \frac{(-1)^n 3^n}{e^{2n}}.$$

   Solution: This is a geometric series. Rewrite the series as:
$$\sum_{n=3}^{\infty} \left(\frac{-3}{e^2}\right)^n = \sum_{n=0}^{\infty} \left(\frac{-3}{e^2}\right)^{n+3} = \sum_{n=0}^{\infty} \left(\frac{-3}{e^2}\right)^3 \left(\frac{-3}{e^2}\right)^n$$
   This is a geometric series with $a = \left(\frac{-3}{e^2}\right)^e$ and $r = \left(\frac{-3}{e^2}\right)$. So the sum is given by
$$s = \frac{a}{1-r} = \left(\frac{-3}{e^2}\right)^2 \left(\frac{1}{1-\left(\frac{-3}{e^2}\right)}\right)$$

2. Is the series
$$\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2}$$
   convergent or divergent? Explain.

   Solution: This can be done by either the comparison test or the integral test. The solution here uses the integral test.

   First note that $\frac{\ln n}{n^2} > 0$ when $n \geq 1$ and that the terms of the series are decreasing.

   We must first find the antiderivative $\int \frac{\ln x}{x^2} dx$. We do this by integration by parts with $u = \ln x$ and $dv = (1/x^2)dx$ so that $du = (1/x)dx$ and $v = -1/x$. Thus
$$\int \frac{\ln x}{x^2} dx = -(1/x)\ln x - \int (-1/x)(1/x)dx = -(1/x)\ln x + [-1/x] + C$$
   So $\int_1^{\infty} \frac{\ln x}{x^2} dx = \lim_{b\to\infty} \int_1^b \frac{\ln x}{x^2} dx = \lim_{b\to\infty} [-(1/x)\ln x + [-1/x]|_1^b = \lim_{b\to\infty} [-(1/b)\ln(b) - (1/b)] - [0-1] = 0+0+0+1$ (by L'Hospital's rule). Since the integral converges, so does the series.

3. Is the series
$$\sum_{n=1}^{\infty} \frac{n}{5^n - 3}$$

convergent or divergent? Explain.

Solution: One way to do this is by the Ratio Test. We compute

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{n+1}{5^{n+1}-3}\frac{5^n-3}{n} = \lim_{n\to\infty} \frac{(n+1)}{n}\frac{5^n-3}{5^{n+1}-3} = \lim_{n\to\infty} \frac{(n+1)}{n}\frac{1/5-3/5^{n+1}}{1-3/5^{n+1}} = 1/5 < 1$$

By the Ratio Test, the series converges.