Quiz 1

September 16

NAME:

JUSTIFY YOUR ANSWERS. Answer each question in the space provided. A correct answer without work shown may be worth 0 points, while an incorrect answer with full justifiction may be worth partial credit. Each question is worth 5 points.

1. Let $a_n = \ln(n+1) - \ln(n)$ for $n \geq 1$. Show that the sequence $\{a_n\}$ is monotonic and bounded, and find the limit.

   Solution:

   (a) **Monotonic:** let $a_n = \ln \frac{(n+1}{n)}$. Then $a_n = f(n)$ for $f(x) = \ln \frac{(x+1}{x)}$. Compute the derivative:

   $$f'(x) = \frac{x}{x+1} \frac{x-(x+1)}{x^2} = \frac{-1}{x(x+1)} < 0$$

   whenever $x > 0$. Since $f'(x) < 0$, the function is decreasing so the sequence is decreasing too.

   (b) **Bounded** Since $\{a_n\}$ is decreasing, it is bounded above by $a_1 = \ln 2$. Since $(n+1)/n = 1+1/n > 1$ when $n > 0$, we have $\ln(n+1)/n > 0$, so $\{a_n\}$ is bounded below by 0.

   (c) **Limit**

   $$\lim_{n\to\infty} \ln \frac{n+1}{n} = \ln(\lim_{n\to\infty} \frac{n+1}{n}) = \ln 1 = 0$$

   since ln is a continuous function.

2. Let $b_n = \frac{[(-1)^n n]+1}{n}$ for $n \geq 1$. Is the sequence $\{b_n\}$ monotonic, bounded, alternating, convergent?

   Solution: Write $b_n = (-1)^n + 1/n$.

   (a) **Alternating** Look at the first few terms:

   $$b_1 = 0 \qquad b_2 = 3/2 \qquad b_3 = -2/3 \qquad \ldots$$

   In general, $b_{2n} > 0$ (since $1 + 1/2n > 0$) and $b_{2n-1} < 0$ (since $-1 + 1/(2n-1) < 0$. Thus the sequence is *eventually alternating* but it is not STRICTLY alternating since $b_1 = 0$.

   (b) **Montonic** The sequence is not monotonic, as the first few terms indicate. In fact, this sequence is not even eventually monotonic since $b_{2n} > b_{2n-1}$ and $b_{2n} > b_{2n+1}$.

   (c) **Bounded** Since $1 + 1/n < 2$ and $-1 + 1/n > -1$, we have $-1 < a_n < 2$ so the sequence is bounded.

   (d) **Divergent** The subsequence b_{2n} converges to 1, but the subsequence b_{2n+1} converges to -1. Thus the limit $\lim_{n\to\infty} b_n$ does not exist.

3. Give an example of a sequence which is alternating and convergent.

   One such example is $a_n = (-1)^n 1/n$. No proof necessary.