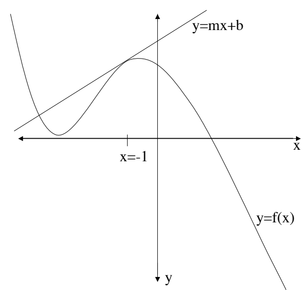
Quiz 3: Friday, October 15
Math 251 Lecture 01

NAME and/or ID: ____

Calculators, books, notes and the help of friends is NOT PERMITTED!
Part I: Multiple choice. Circle the correct answer. There will be no partial marks.

[3 pts]

1. The figure below represents the graph of $y = f(x)$. Which equation below represents the tangent line at $x = -1$?



(a) $y = f(1)x + f'(-1)$;

(b) $y = f(1)x + f(-1) + f'(-1)$;

(c) $y = f'(-1)x + f(-1)$;

(d) $y = f'(-1)x + f(-1) + f'(-1)$.

Solution: The answer is **(d)**.
The slope of the tangent line is given by evaluating the derivative at $x = -1$, that is $f'(-1)$. The point on the curve which the tangent line passes through is $(-1, f(-1))$. Using the point-slope form for the equation of the tangent line, we get

$$y - f(-1) = f'(-1)(x - (-1)) \quad \text{or} \quad y - f(-1) = f'(-1)(x + 1)$$

Putting it into the form we have above, we see the answer is d).

[3 pts]

2. Suppose that $f'(x) = \frac{\sin x}{\cos^2 x} = \sec x \tan x$. Which one of the functions below could be $f(x)$?

(a) $f(x) = \frac{1}{\cos x}$;

(b) $f(x) = \frac{1}{\sin x}$;

(c) $f(x) = \sin x \frac{1}{\cos x}$;

(d) $f(x) = \cos x \frac{1}{\sin x}$.

Solution: The answer is **(a)**.
Using the reciprocal rule, we compute the derivatives:

- $\frac{d}{dx}(\frac{1}{\cos x}) = \frac{-(-\sin x)}{[\cos x]^2} = \sec x \tan x$
- $\frac{d}{dx}(\frac{1}{\sin x}) = \frac{-\cos x}{[\sin x]^2} = -\csc x \cot x$

To compute the other derivatives, we'll use what we computed above plus the multiplicative rule:

- $\frac{d}{dx}(\frac{\sin x}{\cos x}) = \sin x(\sec x \tan x) + (\cos x)\frac{1}{\cos x}$
- $\frac{d}{dx}(\frac{\cos x}{\sin x}) = \cos x(-\csc x \cot x) + (-\sin x)\frac{1}{\sin x}$

Part II. SHOW ALL YOUR WORK. Answer each question in the space provided. A correct answer without work shown may be worth 0 points, while an incorrect answer with work shown may be worth partial credit.

[7 pts]

3. Use the **definition of the derivative** to show that

$$\frac{d}{dx}(\sqrt{x-1}) = \frac{1}{2\sqrt{x-1}}.$$

Solution:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{(x+h)-1} - \sqrt{x-1}}{h} =$$

$$\lim_{h\to 0} \frac{\sqrt{(x+h)-1} - \sqrt{x-1}}{h} \left(\frac{\sqrt{(x+h)-1} + \sqrt{x-1}}{\sqrt{(x+h)-1} + \sqrt{x-1}} \right) = \lim_{h\to 0} \frac{(x+h)-1-(x-1)}{h\left(\sqrt{(x+h)-1} + \sqrt{x-1}\right)}$$

$$= \lim_{h\to 0} \frac{h}{h\left(\sqrt{(x+h)-1} + \sqrt{x-1}\right)} = \lim_{h\to 0} \frac{1}{\sqrt{(x+h)-1} + \sqrt{x-1}} = \frac{1}{\sqrt{x-1} + \sqrt{x-1}}$$

$$= \frac{1}{2\sqrt{x-1}}$$

[7 pts]

4. Let $f(x) = x^2 - 1$ and $g(x) = x^3 + 4$. Show that there is a real number c satisfying $f(c) = g(c)$.

Solution: This is an Intermediate Value Theorem (IVT) problem.
Let $F(x) = f(x) - g(x) = -x^3 + x^2 - 5$. The function $F(x)$ is continuous everywhere because it is a polynomial. We note that

$$F(0) = -5 < 0$$

$$F(-2) = 8 + 2 - 5 = 5 > 0$$

By the IVT, there must be a number c satisfying $-2 < c < 0$ and $F(c) = 0$. If $F(c) = 0$, then $f(c) = g(c)$.