Quiz 1: Thursday, September 23
Math 251 Lecture 01

NAME: _____

SHOW ALL YOUR WORK. Answer each question in the space provided. A correct answer without work shown may be worth 0 points, while an incorrect answer with work shown may be worth partial credit. CALCULATORS ARE NOT PERMITTED.

[3 pts] 1. Which equation corresponds to the graph of $y = \sin(x)$ shifted up by 1 unit and π units to the right?

(a) $y = \sin(x + \pi) + 1$

(b) $y = \sin(x - \pi) + 1$

(c) $y = \sin(x + \pi) - 1$

(d) $y = \sin(x - \pi) - 1$

Solution: Shifting up by 1 unit corresponds to adding 1 to the equation for y. Shifting right by π units corresponds to substituting $x - \pi$ in place of x. Hence, the answer becomes

$$y = \sin(x - \pi) + 1.$$

The answer is b).

[3 pts] 2. Let $f(x) = x^2 + 10$ and $g(x) = \sqrt{x}$. What is the domain of $(f \circ g)(x)$?

(a) $\mathbb{R}$ (all reals)

(b) $\{x : x \geq 0\}$

(c) $(0, \infty)$

(d) The function is undefined.

Solution: Since we are considering the product of the two functions, the domain will be the intersection of the domains of $f(x)$ and $g(x)$. We begin by determining their respective domains:

$\underline{f(x) = x^2 + 10}$: $D_f = \{x : x \in \mathbb{R}\}$, since $f(x)$ is polynomial.

$\underline{g(x) = \sqrt{x}}$: $D_g = \{x : x \geq 0\}$, since we can only take square roots of real numbers that are not negative.

Hence, the domain is:

$$D_{f*g} = \{x : x \geq 0\} = [0, \infty)$$

The answer is b).

[7 pts] 3. Determine the set of real numbers x that satisfy the inequality

$$2|x - 2| \geq x + 1.$$

Solution: We answer this question by appealing to the definition of absolute value.

$\underline{x \geq 2}$: In this case, $x - 2 \geq 0$, so $|x - 2| = x - 2$. Therefore our equation becomes

$$2|x - 2| \geq x + 1 \;\Rightarrow\; 2x - 4 \geq x + 1 \;\Rightarrow\; x \geq 5.$$

We must intersect this with $x \geq 2$ because that was the assumption for this case, but in this case this makes no difference and we have the solution :$x \geq 5$.

$\underline{x < 2}$: In this case, $x - 2 < 0$, so $|x - 2| = -(x - 2)$. Therefore our equation becomes

$$2|x - 2| \geq x + 1 \;\Rightarrow\; -2x + 4 \geq x + 1 \;\Rightarrow -3x \geq -3 \;\Rightarrow\; x \leq 1.$$

Hence, the solutions in the range $x < 2$ is $x \leq 1$.

The complete answer is:

$$\mathbf{x \leq 1} \;\text{ and }\; \mathbf{x \geq 5}.$$

[7 pts] 4. Find the distance between the points $(-1, 2)$ and $(-4, 6)$.

Solution: The distance formula says that the distance between two points (x, y) and (a, b) is

$$d = \sqrt{(x - a)^2 + (y - b)^2}$$

In this case, we have

$$d = \sqrt{(-1 + 4)^2 + (2 - 6)^2} = \sqrt{3^2 + 4^2} = 5.$$