

Name: _____

Final Exam

Directions: You must show all of your work for full credit. A correct answer with no work shown is worth no points, but an incorrect or partial answer with some work shown MAY be worth partial credit. No calculators, books, notes, or collaborating. Divine intervention, however, is permitted.

Some problems require you to complete your choice of 3 out of 5 parts. Please clearly indicate which 3 parts (and ONLY 3 parts) you would like graded by circling those problems. If more than 3 parts are marked, I will give you credit for the 3 LOWEST scoring problems.

1. (30 points) Find $\frac{dy}{dx}$ for 3 of the following 5 functions:

(a) $y = \sec \sqrt{x^2 + 1}$

(b) $y = \frac{x^2+5}{\ln x}$

(c) $x^2 - 2xy + 4y^2 = 3$

(d) $y = \sin^{-1} x$

(e) $y = (\sin x)^x$

2. (30 points) Find 3 of the following 5 limits:

(a) $\lim_{x \rightarrow 2} \frac{x^2 + 2x - 8}{x - 2}$

(b) $\lim_{x \rightarrow \infty} \frac{x + 1}{\sqrt{x^2 + 10}}$

(c) $\lim_{x \rightarrow 0^+} x \ln x$

(d) $\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$

(e) $\lim_{x \rightarrow \infty} \frac{\sinh x}{e^x}$

3. (30 points) Integrate 3 of the following 5 integrals:

(a) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x} dx$

(b) $\int_0^1 x e^{x^2} dx$

(c) $\int \frac{1}{1+9x^2} dx$

(d) $\int \sinh x dx$

(e) $\int_{-1}^1 \sqrt{1-x^2} dx$

Hint: Recognize this as the area of a familiar region.

4. Consider the function $f(x) = \frac{2x^2}{x^2-1} = \frac{2x^2}{(x+1)(x-1)}$.

(a) 5 points. Find the x - and y -intercepts of $f(x)$, if they exist.

(b) 10 points. Find any horizontal and vertical asymptotes of $f(x)$.

(c) 10 points. Find and identify any critical points of $f(x)$. Find the regions where $f(x)$ is increasing or decreasing.

(d) 10 points. Find any inflection points of $f(x)$. Find the regions where $f(x)$ is concave up or concave down.

- (e) 5 points. Sketch the graph of $f(x)$. Be sure to label the intercepts, asymptotes, critical points and inflection points.

5. 15 points. Water is poured into a reservoir in the shape of an inverted cone 6 ft tall with base radius of 4 ft. If the water level is rising at the constant rate of $\frac{1}{2}$ m/sec, how fast is the water being poured in at the instant the depth is 2 ft?
(Volume of a cone = $\frac{1}{3}\pi r^2 h$. Hint: Don't forget how to use similar triangles.)

6. 10 points. Remember that the fluid force on a *horizontal* plate lying underwater is given by density \times depth \times area.

A *vertical* dam in the shape of an inverted isosceles triangle 6ft high and with an 8 ft base bounds a reservoir. Set up, but DO NOT EVALUATE an integral computing the force of the water on the dam when the water is 5 ft deep.

(The density of water is $\sigma = 62.5\text{lbs/ft}^3$.)

7. 10 points. Find the average value of the function $3 - x^2$ on $0 \leq x \leq \sqrt{3}$.

8. 10 points. The half life of ^{14}C is approximately 5700 years. The Dead Sea Scrolls are approximately 2000 years old. What fraction of the original ^{14}C remains in them?

9. 10 points. Does the function $y = e^{2x} + 3$ have an inverse? If so, find it. If not, why not?

10. 10 points. Show that $y = A \cosh cx + B \sinh cx$ satisfies the differential equation

$$y'' - c^2 y = 0.$$