Name:_____

Lab Section:_____

Exam Code: Math 251 L04 and L06/13

Calculators, books, notes and collaboration are *not permitted.*

120 MINUTES

UNIVERSITY OF CALGARY

FINAL EXAM

SESSION 2003
FALL SEMESTER

DATE: 22 December 2003
TIME: 3:30 - 5:30

Please write your student number on every page, beginning with this one.
Good luck!

For Markers Only	
1	/5
2	/5
3	/15
4	/10
5	/5
6	/5
7	/15
B	/40
Σ	/100

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SECTION A

Show your work. An incorrect answer with work shown may receive partial credit, while correct answers with no work may receive no credit.

[10]

1 Find $\frac{dy}{dx}$ for $y = (2 + \cos(x))^{\ln(2+\cos(x))}$.

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[6] 2 Let $f(x)$ be differentiable on $[a, b]$. If $f'(x) > 0$ for every $x \in [a, b]$, explain why

$$\frac{f(b) - f(a)}{b - a} > 0.$$

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3 Let $f(x) = \frac{x^3-1}{x^2}$. The first two derivatives of $f(x)$ are:

$$f'(x) = \frac{x^3+2}{x^3}, \qquad f''(x) = \frac{-6}{x^4}.$$

[5] (a) Find all asymptotes of $f(x)$.

[3] (b) Find all x and y coordinates of the intercepts, critical points, and inflection points of $f(x)$, if they exist.

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[5] (c) Find all of the intervals where $f(x)$ is increasing and all intervals where $f(x)$ is decreasing.

[5] (d) Find all of the intervals where $f(x)$ is concave up and all intervals where $f(x)$ is concave down.

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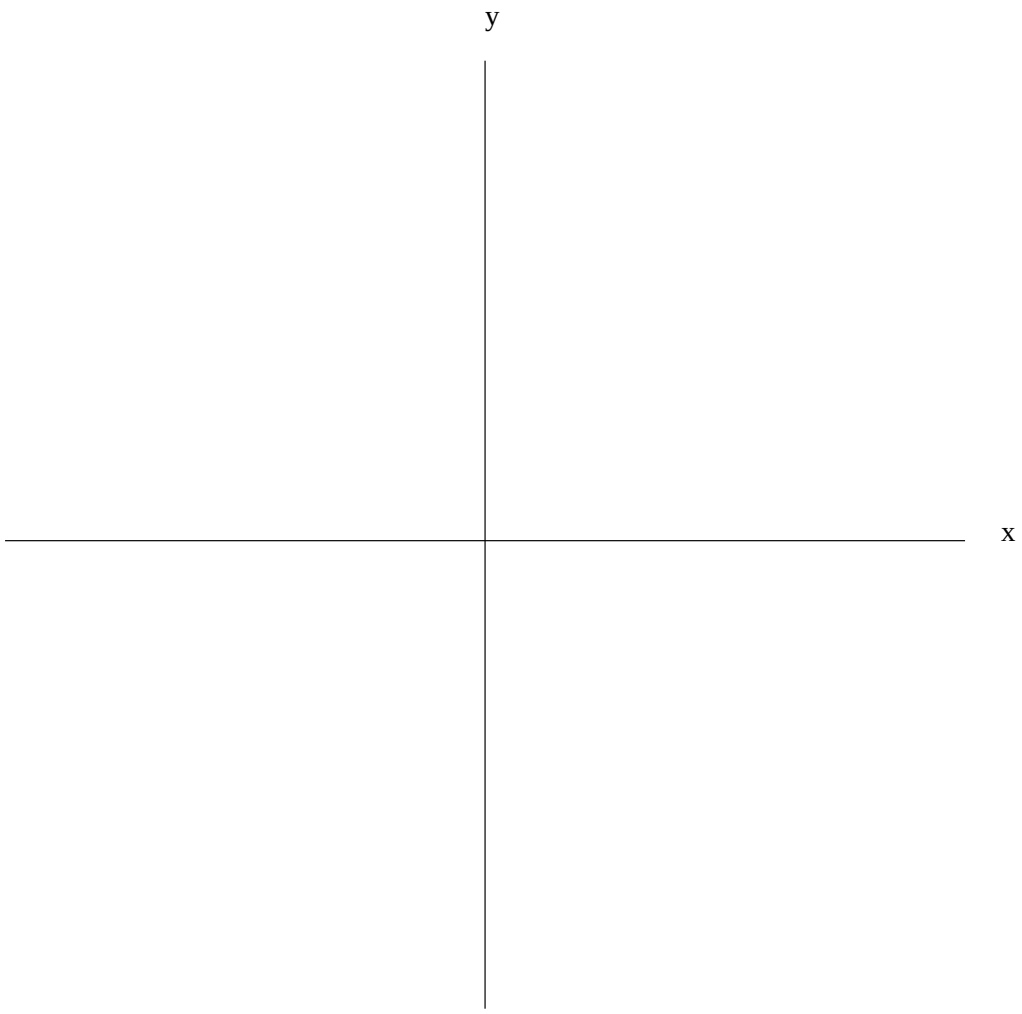
[2] (e) Sketch the graph of $f(x)$ on the axes below.



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[10]

4 Special containers are manufactured, each having a square base, four identical sides, and no top. Each container holds a volume of 4000 cm^3. Find the container size having minimal cost. That is, find the length b of one side of the base and the height h of the sides such that the total surface area of the base and sides of the container is a minimum.

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5 Evaluate the integrals:

[5] (a) $\int \frac{2x+1}{(x+1)^{3/2}} dx =$

[5] (b) $\int \tan^2(x) \sec^2(x) dx =$

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SECTION B

Circle the correct answer. Each one is worth 3 points.

6 For which value of k is the following function continuous

$$g(x) = \begin{cases} 2kx^2 - 1 & x \leq 1 \\ x^3 + kx & x > 1 \end{cases}$$

(a) $k = 0$

(b) $k = 1$.

(c) $k = 2$.

(d) this function is never continuous.

7 If $f(x)$ is continuous at $x = 2$, then

(a) $f(2) = 2$.

(b) for every $\epsilon > 0$ and for any value of x, $|f(x) - 2| < \epsilon$.

(c) for every $\epsilon > 0$ you can find a value of x near $x = 2$ with $|f(x) - 2| < \epsilon$.

(d) for every $n > 1$, you can find a value of x near $x = 2$ with $|f(x) - f(2)| < 1/n$.

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8 If the limit

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

exists and is equal to a positive number, then

(a) f has a critical point at x.

(b) f has an asymptote at x.

(c) f is extremized at x.

(d) f is increasing at x.

9 Compute

$$\lim_{x \to infty} \frac{\sqrt{4x^2 - x + 3}}{3x + 1}$$

(a) $3\sqrt{3}$.

(b) 3

(c) 2/3.

(d) 4/3.

(e) Does not exist.

10 The limit $\lim_{x \to 0} \frac{\ln(x+1)}{\sin(x)}$ is:

(a) 0.

(b) 1.

(c) e/π.

(d) Does not exist, but tends to $-\infty$.

(e) Does not exist, but tends to ∞.

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11 The equation $(x+1)(e^{(x+2)}+1)=0$ has

(a) no solutions.

(b) only one solution.

(c) exactly two solutions.

(d) more than two solutions.

12 If the function $f(x)$ is decreasing on $(-\infty,\infty)$, then

(a) $f(x)<0$ for all x.

(b) $f'(x)<0$ for all x.

(c) $f(x)$ can not have a local extremum.

(d) $f(x)$ can not have an inflection point.

13 For $y(x-y)=x^2+y$ find y'.

(a) $y'=(2x-y)/(x-2y-1)$.

(b) $y'=(x^2+y)/(x-y)$.

(c) $y'=(2x+y)/(x-y)$.

(d) $y'=(x-2y)/(2y-x+1)$.

(e) $y'=(y-2x)/(2y)$.

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14 Classify the critical points of $f(x) = x^3 + x^{-1}$.

(a) f has one local min, and one local max.

(b) f has three extreme points.

(c) f has one global min, and one global max.

(d) f has two critical points, but no extreme points.

(e) f has no critical points.

15 A ten foot ladder is leaning against a vertical wall. If the foot of the ladder is being pulled away at a speed of 2 ft/s, how fast is the top of the ladder moving when it is still 6 feet off the ground?

(a) 8/3 ft/s.

(b) 2 ft/s.

(c) 3 ft/s.

(d) 4/3 ft/s.

(e) 3 ft/s.

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16 Let f be the function defined by $f(x) = x^2$. A Riemann sum of f based on a partition of the closed interval $[3, 5]$ will have exactly four terms if

(a) the subintervals of the partition are defined by the set of points $x_0 = 3$, $x_1 = 4$, $x_2 = 5$.

(b) the subintervals of the partition are defined by the set of points $x_0 = 3$, $x_1 = 7/2$, $x_2 = 4$, $x_3 = 9/2$, $x_4 = 5$.

(c) the sum of the areas of the rectangles generated by the partition equals the area bounded by $y = x^2$, $y = 0$, $x = 3$ and $x = 5$.

(d) the partition generates 4 rectangles whose area adds up to $\int_3^5 x^2 dx$.

17 If the function f is continuous everywhere then $\frac{d}{dx} \int_a^x f(t)\, dt$ is always

(a) a positive number.

(b) $f(x) - f(a)$.

(c) $f(x)$.

(d) $F(x) - F(a)$, where $F'(x) = f(x)$.

(e) an antiderivative of f.

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18 $\int_1^2 \ln(x)\, dx$ is always

(a) positive.

(b) negative.

(c) zero.

(d) nonexistent.

END OF EXAM