Take Home Quiz 2: Riemann Sums
Due Wednesday, December 8
Math 251 Lecture 01

You will have only one chance to complete this quiz. Good luck!

[5 pts]

1. Evaluate the sum

$$\sum_{i=0}^{n} i^2 + 2i + 1$$

in terms of n. (Hint: don't forget that your sum formulas apply to sums which begin at $i = 1$.)

First we'll get rid of those pesky 0-terms in the sum:

$$\sum_{i=0}^{n} i^2 + 2i + 1 = 0 + 0 + 1 + \sum_{i=1}^{n} i^2 + 2i + 1 = 1 + \sum_{i=1}^{n} i^2 + 2\sum_{i=1}^{n} i + \sum_{i=1}^{n} 1$$

Now apply the sum formulas (from class notes or from your textbook): $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ and $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ and $\sum_{i=1}^{n} 1 = n$. Then,

$$\sum_{i=0}^{n} i^2 + 2i + 1 = 1 + \frac{n(n+1)(2n+1)}{6} + 2\frac{n(n+1)}{2} + n.$$

From here, there are various ways to simplify the sum if you would like to.

[5 pts]

2. Express the limit below as a definite integral, but do not evaluate it.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi}{n} \sin\left(\frac{\pi i}{n}\right)$$

Because the sum starts with the term $i = 1$, we might suspect that this is Riemann sum using the right hand method. If it is, then the definition of the integral says:

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{n=1}^{\infty} f(x_i)\Delta x$$

where $\Delta x = \frac{b-a}{n}$ and $x_i = a + \frac{b-a}{n} i$. In this case, it looks like the function $f(x)$ is $\sin(x)$. That means that $x_i = \frac{\pi i}{n}$, so $a = 0$ which means $b = \pi$. This also goes with $\Delta x = \pi/n$. So, the integral should be:

$$\int_0^{\pi} \sin(x)dx.$$

[5 pts]

3. Find the area bounded by $x = 2$, $x = 5$, $y = 2x^2 + 2$ and the $x - axis$ by computing an appropriate Riemann sum (you may NOT use the Fundamental Theorem of Calculus as a short cut).

I prefer the right endpoint method, so I'll use that. As an integral, this areas is computed by

$$\int_2^5 2x^2 + 2\ dx.$$

Now we have to use a Riemann sum to compute the integral. The format for the right-hand endpoint method is also in the last solution, it's

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

where $\Delta x = \frac{5-2}{n} = 3/n$ and $x_i = 2 + (3/n)i$. Plugging these into the equation above, we have:

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(2 + (3/n)i)(3/n) = \lim_{n\to\infty} \sum_{i=1}^{n} (2(2 + 3i/n)^2 + 2)(3/n)$$

. Now we have to simplify this, so we can use the sum formulas above. There are lots of choices for what to do to simplify here ... do what you are most comfortable with. Here is one way:

$$\lim_{n\to\infty} (6/n) \sum_{i=1}^{n} (2 + 3i/n)^2 + 1 = \lim_{n\to\infty} (6/n) \sum_{i=1}^{n} 4 + 12i/n + 9i^2/n^2 + 1$$

$$= \lim_{n\to\infty} (6/n)(5 \sum_{i=1}^{n} 1 + 12/n \sum_{i=1}^{n} i + 9/n^2 \sum_{i=1}^{n} i^2)$$

$$= \lim_{n\to\infty} (6/n)(5n + \frac{12}{n}\frac{n(n+1)}{2} + \frac{9}{n^2}\frac{n(n+1)(2n+1)}{6})$$

$$= \lim_{n\to\infty} 30 + 36\frac{n+1}{n} + 9\frac{n+1}{n}\frac{2n+1}{n}$$

$$= 30 + 36 + 18 = 84.$$

So the area is 84 square units.