Take Quiz 2
Due Wednesday, November 6
Math 251 Lecture 01

You will have one more attempt to improve your score on this quiz. Each attempt replaces the previous quizzes. Good luck!

[5 pts]

1. Let $f(x) = 2x - 1$, $a = 0$ and $L = -1$. Find a number $\delta > 0$ such that if $|x - a| < \delta$, then $|f(x) - L| < 0.5$.

   Let's try $\delta = 0.25$. If $|x - 0| = |x| < 0.25$, then

   $$|2x - 1 - (-1)| = |2x| = 2|x| < 2(0.25) = 0.5.$$

   That is, the value $\delta = 0.25$ guarantees that $|f(x) - L| < 0.5$.

[5 pts]

2. Use the formal definition of the limit to verify:

   $$\lim_{x\to 0} x^2 = 0.$$

   Let $\epsilon > 0$. Let $\delta < \sqrt{\epsilon}$. If $|x| < \delta$, then

   $$|x^2| = |x|^2 < (\delta)^2 < (\sqrt{\epsilon})^2 = \epsilon.$$

   This verifies that $\lim_{x\to 0} x^2 = 0$.

[5 pts]

3. If $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$ prove that $\lim_{x\to a} f(x)g(x) = LM$. For hints, see number 33 on page 94.

   Let $\epsilon > 0$. Since $\lim_{x\to a} f(x) = L$, there exists a $\delta_1 > 0$ with $|f(x) - L| < \epsilon/2(1 + |M|)$ whenever $|x-a| < \delta_1$. Since $\lim_{x\to a} g(x) = M$, there's a $\delta_2 > 0$ with $|g(x)-M| < \epsilon/2|L|$. We'll use the fact that $\lim_{x\to a} g(x) = M$ again. Let $\delta_3 > 0$ be such that $|g(x) - M| < 1$ when $|x - a| < \delta_3$. This implies that $|g(x)| < |M| + 1$. You can quote this fact from the textbook exercise, or you can prove it:

   $$|g(x)| = |g(x) - M + M| \leq |g(x) - M| + |M| < 1 + |M|.$$

   Now, let δ be the smallest of $\delta_1, \delta_2, \delta_3$. One often writes $\delta = min(\delta_1, \delta_2, \delta_3)$. When $|x - a| < \delta$, we can use all of the inequalities we discovered above.

   Using the hint in the textbook, we have:

   $$|f(x)g(x)-LM| \leq |g(x)||f(x)-L|+|L||g(x)-M| < (1+|M|)\frac{\epsilon}{2(1 + |M|)}+|L|\frac{\epsilon}{2|L|} = \epsilon.$$

   This verifies that $\lim_{x\to a} f(x)g(x) = LM$.