AMAT 483

Introduction to Computational Finance

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University of Calgary

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Course information

- website: http://math.ucalgary.ca/~aware/amat483
- text: Financial Option Valuation by Des Higham, who maintains an author site with some more helpful information.
- assessment: 4 assigments (40%), 1 midterm (20%), 1 final (40%)
- office hours: TBA (MS586)

Overview

- Review of options and derivative valuation
- Asset price models: from binomial trees to geometric Brownian motion
- Simulating asset prices
- Black-Scholes option pricing
- Option hedging and greeks
- Risk-neutrality
- Implied volatility and historical volatility
- The binomial method
- The Monte Carlo method
- Exotic options
    - Cash-or-nothing options
    - American options
    - Path-dependent options
- Finite difference methods

Financial assets

There is a dizzying variety of financial assets being traded in public exchanges such as the TSX, the NYSE, NASDAQ, the CBOE or the London Stock Exchange.

Two important properties of assets are their liquidity and the ease with which they may be stored.

- **Stocks** represent part ownership in a company. Owners of stocks may receive dividend payments or a share in the proceeds if the company is sold. [AAPL]
- **Bonds** are promises to pay $1 at some future date, possibly together with fixed interest payments ('coupons') at regular intervals. They might be issued by governments [Cdn. bonds (T-bills, fixed-income bonds)] or by companies [MarketWatch].
- **Indexes** are baskets of stocks or bonds, may be more liquid and less volatile than individual stocks. [DJIA]
- **FX** - foreign exchange markets - deal with various kinds of contracts involving two or more currencies. [CAD:USD,CME CAD futures]

Financial assets

Commodities are physical assets, so that transportation and storage costs become important, and there may be tangible convenience associated with ownership. [CBOT wheat, NYMEX natural gas]

Credit instruments of various forms experienced explosive market growth over the last decade, but have caused some trouble.

Futures/options are derivative assets, whose value depends on a set of underlying assets. [AAPL]

Derivatives

A derivative is a financial instrument whose value depends on (or derived from) the values of other more basic underlying variables.
Examples of "underlying variables": the price of a stock, amount of snow falling at a certain region
Examples of derivatives: futures, forwards, options, swaps

Markets and traders

Exchange markets

- individuals trade standardized contracts that have been defined by the exchange
- examples: Chicago Mercantile Exchange, CBOT, CBOE
- margin requirement + marking to market → eliminate credit risk (the risk that the contract will not be honoured)

Over-the-counter markets

- phone/computer linked network of dealers → no physical location
- trades done over the phone, usually between two financial institutions or between a financial institution and one of its clients
- telephone conversations are usually taped → avoid dispute
- terms of contract not specified by an exchange → participants can negotiate any mutually attractive deal
- credit risk exists

Hedgers

use derivatives to reduce risk that they face from potential future movements in market variables.

Speculators

use derivatives to bet on future direction of a market variable.

Arbitrageurs

take offsetting positions in two or more instruments to lock in a profit.

Forward contract

A **forward contract** is an agreement between two parties to buy or sell an asset at a certain time in future (**delivery date/maturity**) for a certain price (**future price/delivery price**). Forward contracts differ slightly from futures contracts, which are normally traded on an exchange. The party who agrees to buy the underlying asset is taking a *long position*; the other party who agrees to sell the underlying asset is taking a *short position*. It costs nothing for anyone to enter into a forward contract, i.e. *the price of the forward contract is zero*.

Forward contract

Example (long position)

On January 1, the forward price of gold per ounce is \$400, to be delivered in June. Contract size is 100 ounces. If investor ABC buys two such contracts, (s)he agrees to buy 200 ounces in June at a price of \$400 per ounce. (S)he is now taking a long position.

- **Scenario 1:** Suppose that the price of gold per ounce is \$420 in June. Then ABC could use \$80000 (=\$400 $\times$ 200) to buy 200 ounces rather than using \$84000 (=\$420 $\times$ 200). The profit-and-loss (P/L) is \$84000-\$80000=\$4000.
- **Scenario 2:** Suppose that the price of gold per ounce is \$380 in June. Then ABC is obligated to use \$80000 (=\$400 $\times$ 200) to buy 200 ounces rather than using \$76000 (=\$380 $\times$ 200). The P/L is \$76000-\$80000=-\$4000.

Forward contract

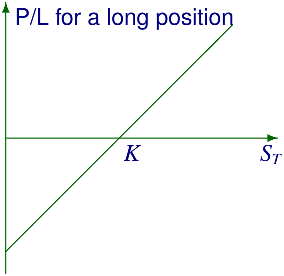
Example (short position)

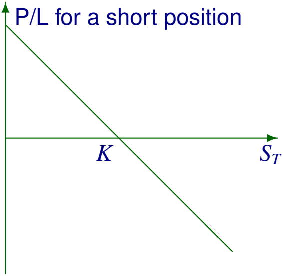
On January 1, the forward price of gold per ounce is \$400, to be delivered in June. Contract size is 100 ounces. If investor DEF sells three such contract, (s)he agrees to sell 300 ounces in June at a price of \$400 per ounce. (S)he is now taking a short position.

- Scenario 1: Suppose that the price of gold per ounce is \$420 in June. Then DEF is forced to sell 300 ounces of gold at \$120000 (=\$400 $\times$ 300) rather than at \$126000 (=\$420 $\times$ 300). The P/L is \$120000-\$126000=-\$6000.
- Scenario 2: Suppose that the price of gold per ounce is \$380 in June. Then DEF could sell 300 ounces of gold at \$120000 (=\$400 $\times$ 300) rather than at \$114000 (=\$380 $\times$ 300). The P/L is \$120000-\$114000=+\$6000.

Forward contract

- In general, the P/L from a long position in a forward contract on one unit of an asset is $S_T - K$, where K is the forward price and S_T is the spot price of the asset at the delivery date.
- In general, the P/L from a short position in a forward contract on one unit of an asset is $K - S_T$, where K is the forward price and S_T is the spot price of the asset at the delivery date.





Closing out forward positions

The vast majority of forward contracts do not lead to actual physical delivery. Most traders choose to close out their positions prior to the delivery date. Closing out a position means entering into the opposite type of trade from the original one.

Example

A New York investor has bought one July corn forward contract on March 5; the forward price on March 5 is $5 per unit.
On April 20, the forward price of the July corn forward contract is $5.60 per unit. The investor sells one July corn forward contract on that day.
The long position made on March 5 is now cancelled by the short position made on April 20, and a guaranteed profit of $0.60 (=$5.60-$5) per unit will be made (in July).

A note on present value

- Cashflows taking place *at different times* cannot be directly compared. In the previous example, the $0.60 to be received in July is not the same as $0.60 in April.
- If I have $20 today, I can put it in an interest-bearing account, and the account will (typically) have more than $20 in a year's time.
- A zero-coupon bond is a contract that promises to pay a specified amount at some point in the future. Putting my money in an interest-bearing account (with no possibility of early withdrawal) is equivalent to investing it in zero-coupon bonds.
- For example, if a zero-coupon bond paying out $1 in a year's time costs $0.80 today, then my $20 will buy 25 of these bonds today. In one year they will pay out $25.
- Present (or future) value can be computed by using zero-coupon bonds. For example, if B_0^T is the price today of a bond paying $1 at time T, then B_0^T is the present value of $1 at time T, and $1/B_0^T$ is the future value at time T of $1 today.

The forward price in an ideal market

Reproducing the payoff from a forward contract

Suppose that the forward price (for expiry time T) for an underlying asset S at time 0 is K. If we construct a portfolio Π at time 0 by purchasing the asset (at a cost of S_0) and selling K zero-coupon bonds (receiving KB_0^T), then the value of our portfolio at time T is

$$\Pi_T = S_T - K.$$

This is the same as long position in the forward contract.

Using arbitrage to determine K

If there is no cost associated with holding on to the asset, and no risk of default from either party, then we have two positions at time 0 that in all states of the world will produce the same cashflow at time T.

There will be arbitrage unless the values of these positions at time 0 are the same. Since the forward contract has zero value when it is entered in to, this means that $\Pi_0 = 0$, i.e.

$$S_0 = KB_0^T, \quad \text{so that} \quad K = \frac{S_0}{B_0^T}.$$

Call and put options

Call option

A call option gives the holder the **right** to buy an asset for a set price (the strike price K). But, in contrast with the forward or futures contract, there is no *obligation* on the holder.

Terminology

Exercise	- when the holder buys the asset under the terms of the option.
Strike price	- the price that will be paid for the asset on exercise.
Expiry time	- the termination of the contract.
Exercise style	- European (only at the expiry time), American (at any time prior to the expiry time), Bermudan (at some set times prior to the expiry time).
Premium	- what must be paid to purchase the option contract today.
Payoff	- effective cashflow on exercise.

Call option

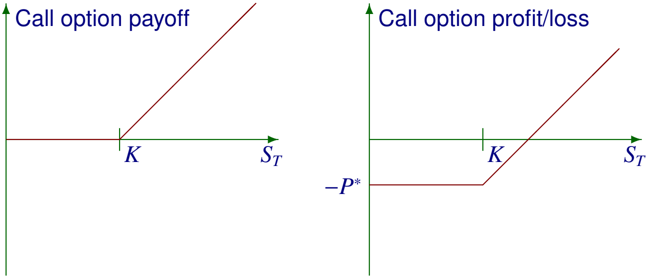
A call option gives the holder the **right** to buy an asset for a set price (the strike price K). But, in contrast with the forward or futures contract, there is no *obligation* on the holder.

Example

Company A's stock is presently at \$25/share. A European call option struck at \$26, expiring in one year's time, costs \$1.50 today.

- If the stock price has risen to \$30 at that time, an investor who purchased this option today will exercise the option and receive a payoff worth \$4; they will have made a profit of \$2.50 (ignoring the time value of money).
- If the stock price has instead fallen below \$26, the investor will not exercise the option - it will expire worthless. The initial cost of the option will mean the investment has lost \$1.50.

Call option payoff/profit diagrams—for the holder



- The payoff of the call option is

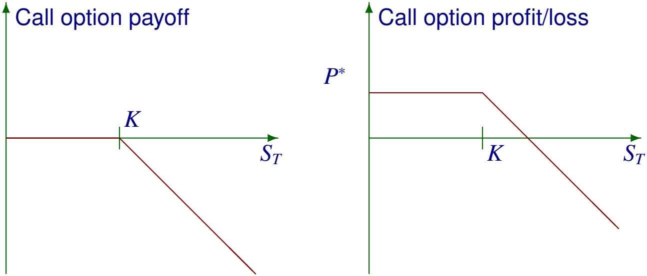
$$(S_T - K)_+.$$

- The profit/loss (at time T) from the call option is

$$(S_T - K)_+ - P^*,$$

where $P^* = P/B_0^T$, and P is the initial premium.

Call option payoff/profit diagrams—for the writer



- For the option **writer**, the payoff of the call option is

$$-(S_T - K)_+.$$

- The negative payoff reflects the fact that the holder has all the rights and none of the obligations, but that it is the other way around for the writer.
- The profit/loss from the call option is

$$P^* - (S_T - K)_+.$$

Put option

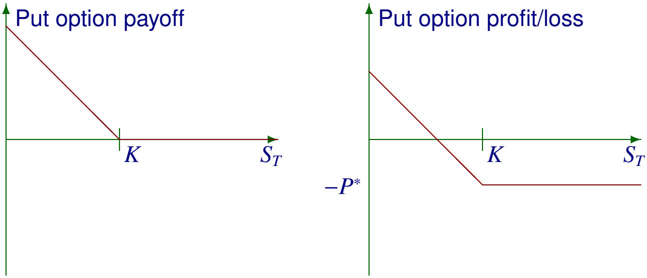
A put option gives the holder the **right** to *sell* an asset for a set price (the strike price K). Again, in contrast with the forward or futures contract, there is no *obligation* on the holder.

Example

Company A's stock is presently at \$25/share. A European put option struck at \$26, expiring in one year's time, costs \$2 today.

- If the stock price has fallen to \$22 at that time, an investor who purchased this option today will exercise the option and receive a payoff worth \$4; they will have made a profit of \$2 (ignoring the time value of money).
- If the stock price has instead risen above \$26, the investor will not exercise the option - it will expire worthless. The initial cost of the option will mean the investment has lost \$2.

Put option payoff/profit diagrams—for the holder



- The payoff of the put option is

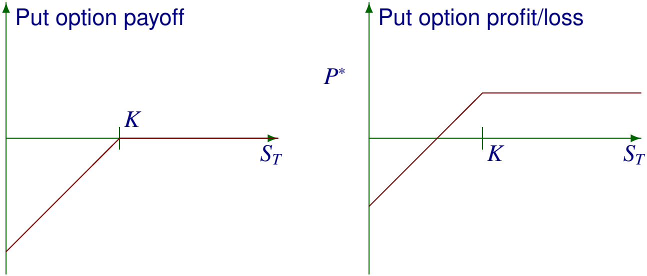
$$(K - S_T)_+.$$

- The profit/loss (at time T) from the put option is

$$(K - S_T)_+ - P^*,$$

where $P^* = P/B_0^T$, and P is the initial premium.

Put option payoff/profit diagrams—for the writer



- For the option **writer**, the payoff of the put option is

$$-(K - S_T)_+.$$

- The negative payoff reflects the fact that the holder has all the rights and none of the obligations, but that it is the other way around for the writer.
- The profit/loss from the put option is

$$P^* - (K - S_T)_+.$$

About options

Options are risky!

- So risky that they were banned in the UK in the 1700s.
- They were only legalised in the US seventy-five years ago.

Leverage

Suppose that you have $150 to invest in company A (whose stock is presently at $25/share). You believe the stock is going to rise over the coming year, and you want to invest your money based on that belief.
Consider the following two strategies:

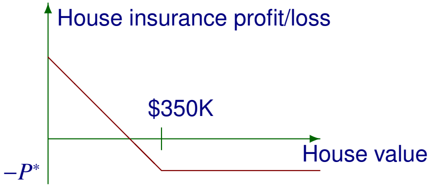
- You can buy six shares. If the stock rises to $30, you make 20% profit. If the stock falls to $20, you lose 20%.
- You can buy one hundred call options struck at $26 (costing $1.50 each). If the stock rises to $30, each option makes you $4, so you make 400% profit! But, if the stock falls to $20, you lose everything.

Options are insurance against risk!

Homeowner's insurance

Suppose you own a $400,000 house, and you purchase an insurance policy against loss of value that might arise if the house is damaged in some way. If the policy has a deductible, you will be responsible for the first $50,000 (say) of any loss, but after that the policy will make good the loss. At the outset, you have to pay the cost (the premium) for this insurance.

The profit/loss as a function of the eventual house value is the same as for a put option!



The major difference between the two scenarios is the ownership of the underlying asset. The homeowner already owns the house—and therefore has a long position in the underlying asset. Buying insurance (buying a the put option) is a partial hedge against this position.

More on bonds and interest rates

- Suppose that at any point in time t we have a series of (zero-coupon) bond contracts on the market with prices $B_t^{T_1}, B_t^{T_2}, \ldots$.
- If the prices are such that the ratio

$$\frac{B_t^{T_i}}{B_t^{T_j}}$$

depends only on the difference $T_i - T_j$, then we may suppose the existence of a constant underlying interest rate r such that

$$B_t^T \propto \mathrm{e}^{-rT}.$$

- Since we must have $B_t^t = 1$ for all t, then we have

$$B_t^T = \mathrm{e}^{-r(T-t)}.$$

More on options

Varieties of options

- American options allow for exercise at any point prior to expiry (European options may be exercised only at expiry).
- Bermudan options allow for exercise at a specified set of dates.
- Barrier options may only kick in if the underlying asset crosses some specified level before the option expires. Alternatively, they may only have value if this does not happen.
- Asian options have payoffs that depend on the average price of the underlying.

Other considerations

Dividends - stock holders get them but option holders don't

Exercise - how and when?

Margins - option writers must post them to protect against default risk

Taxes - ask a professional

Market prices for all kinds of financial assets, including options, are widely available on the web. Here are some numbers from Yahoo:

Apple Inc. (AAPL) At 12:03PM ET: **210.3898** ↓ 1.5902 (0.75%)

Options

View By Expiration: **Jan 10** | Feb 10 | Apr 10 | Jul 10 | Jan 11 | Jan 12

Options Expiring Fri, Jan 15, 2010

Calls							Strike Price	Puts						
Symbol	Last	Change	Bid	Ask	Volume	Open Int		Symbol	Last	Change	Bid	Ask	Volume	Open Int
AAQAX.X	190.50	0.00	186.90	188.10	9	1,113	22.50	AAQMX.X	0.02	0.00	N/A	0.01	1	3,011
AAQAE.X	188.00	0.00	184.40	185.60	9	1,177	25.00	AAQME.X	0.01	0.00	N/A	0.01	10	12,161
AAQAF.X	185.01	0.00	179.40	180.60	17	779	30.00	AAQMF.X	0.01	0.00	N/A	0.01	1	4,735
AAQAG.X	179.36	0.00	174.40	176.45	1	545	35.00	AAQMG.X	0.01	0.00	N/A	0.01	247	1,743
AAQAH.X	173.08	0.00	169.35	170.70	12	411	40.00	AAQMH.X	0.02	0.00	N/A	0.01	1	1,983
QAAAI.X	165.95	0.00	164.40	165.60	1	456	45.00	QAAMI.X	0.03	0.00	N/A	0.01	10	3,337
QAAAJ.X	164.40	0.00	159.40	161.45	2	230	50.00	QAAMJ.X	0.02	0.00	N/A	0.01	72	4,836
QAAAK.X	158.45	0.00	155.15	155.60	32	223	55.00	QAAMK.X	0.02	0.00	N/A	0.01	134	1,701
QAAAL.X	152.70	0.00	149.40	150.60	2	185	60.00	QAAML.X	0.03	0.00	N/A	0.01	153	2,748
QAAAM.X	148.10	0.00	144.40	145.60	9	223	65.00	QAAMM.X	0.01	0.00	N/A	0.01	1	15,532
QAAAN.X	144.20	0.00	139.40	140.60	3	1,298	70.00	QAAMN.X	0.01	0.00	N/A	0.01	20	13,241
QAAAO.X	120.00	0.00	135.10	135.60	2	425	75.00	QAAMO.X	0.01	0.00	N/A	0.01	14	14,440
QAAAP.X	133.00	0.00	129.40	130.60	2	1,488	80.00	QAAMP.X	0.03	0.00	N/A	0.02	13	18,788
QAAAQ.X	130.00	0.00	124.40	125.60	1	6,943	85.00	QAAMQ.X	0.03	0.00	N/A	0.02	3	9,071
QAAAR.X	120.10	↓ 0.98	119.40	121.05	1	6,035	90.00	QAAMR.X	0.01	0.00	N/A	0.02	1	13,394

Prices of options on the market: IBM

International Business Machines Corp. (IBM) At 12:04PM ET: **129.54** ↓ 1.31 (1.00%)

Options

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Options Expiring Fri, Jan 15, 2010

Calls							Strike Price	Puts						
Symbol	Last	Change	Bid	Ask	Volume	Open Int		Symbol	Last	Change	Bid	Ask	Volume	Open Int
IBMAV.X	63.50	0.00	73.70	75.00	52	3	55.00	IBMMV.X	0.05	0.00	N/A	0.02	322	1,815
IBMAL.X	69.30	0.00	68.70	70.05	5	8	60.00	IBMML.X	0.02	0.00	N/A	0.03	2	4,000
IBMAM.X	63.10	0.00	63.70	65.00	25	25	65.00	IBMMM.X	0.03	0.00	N/A	0.03	199	4,933
IBMAN.X	61.80	0.00	58.70	60.00	3	8	70.00	IBMMN.X	0.01	0.00	N/A	0.03	3	4,684
IBMAO.X	42.50	0.00	53.70	54.70	1	1	75.00	IBMMO.X	0.02	0.00	N/A	0.03	112	8,204
IBMAP.X	49.65	0.00	48.70	49.70	8	53	80.00	IBMMP.X	0.01	0.00	N/A	0.03	11	8,250
IBMAQ.X	44.74	0.00	43.70	44.75	1	230	85.00	IBMMQ.X	0.03	0.00	N/A	0.03	21	12,294
IBMAR.X	39.22	↓ 1.27	38.70	39.80	10	867	90.00	IBMMR.X	0.04	0.00	N/A	0.03	21	8,566
IBMAS.X	35.25	0.00	34.45	34.65	10	430	95.00	IBMMS.X	0.02	0.00	0.01	0.03	5	8,000
IBMAT.X	29.20	↓ 1.40	29.50	29.65	25	11,225	100.00	IBMMT.X	0.03	0.00	0.01	0.03	2	12,214
IBMAA.X	24.35	↓ 1.50	24.50	24.65	18	4,919	105.00	IBMMA.X	0.02	↓ 0.01	0.02	0.03	98	9,676
IBMAB.X	19.59	↓ 1.06	19.50	19.65	96	6,584	110.00	IBMMB.X	0.02	↑ 0.01	N/A	0.02	212	12,344
IBMAC.X	14.29	↓ 1.46	14.50	14.65	136	4,164	115.00	IBMMC.X	0.03	↑ 0.01	0.02	0.03	343	10,335
IBMAD.X	9.30	↓ 1.49	9.50	9.65	159	20,907	120.00	IBMMD.X	0.03	↓ 0.01	0.03	0.05	80	10,337
IBMAE.X	4.70	↓ 1.20	4.65	4.75	683	11,914	125.00	IBMME.X	0.15	↑ 0.02	0.14	0.16	1,477	17,108
IBMAF.X	1.00	↓ 0.66	0.99	1.01	3,349	27,375	130.00	IBMMF.X	1.45	↑ 0.57	1.43	1.45	3,985	12,885

Some things we can say about option prices

Let C_0^T denote the price of a call option at time 0 with expiry time T, struck at K, with the price of the underlying at S, and let P_0^T denote the price of the corresponding put option. If we want to avoid *arbitrage*, we have the following bounds.

$$C_0^T \geq 0$$

$$C_0^T \leq S$$

$$C_0^T \geq S - KB_0^T$$

This means that $C(T) \to S$ as $T \to \infty$.

$$P_0^T + S = C_0^T + KB_0^T$$

This is known as put-call parity.