14.2: Single-Population Inferences

- The **sign test** provides inferences about population *medians*, or *central tendencies*, when skewed data or an outlier would invalidate tests based on normal distributions.

14.2: Single-Population Inferences

- One-tailed test for a Population Median, η

  $H_0: \eta = \eta_0$

  $H_a: \eta > \eta_0$ [or $H_a: \eta < \eta_0$]

  Test Statistic:

  S = number of sample measurements greater than [less than] η_0

- Two-tailed test for a Population Median, η

  $H_0: \eta = \eta_0$

  $H_a: \eta \neq \eta_0$

  Test Statistic:

  S = larger of S_1 and S_2 where S_1 is the number of measurements less than η_0 and S_2 the number greater than η_0

14.2: Single-Population Inferences

- One-tailed test for a Population Median, η

  Observed significance level:

  p-value = $P(X \geq S)$

- Two-tailed test for a Population Median, η

  Observed significance level:

  p-value = $2P(X \geq S)$

where X has a binomial distribution with parameters *n* and *p* = 0.5.

Reject H_0 if the p-value is $\leq \alpha$.

Starbucks Coffee

On 6 consecutive days, you purchased the same 16-ounce coffee from a Starbucks shop and determined the caffeine levels:
564 498 259 303 300 307 (ml)

Does the median caffeine level exceed 300 ml? (use alpha = 0.05 level)

You can test H_0: $\eta = 300$ vs H_a: $\eta > 300$. Of the 6 measurements, one = 300 and we take it out. For the remaining 5, 4 are > 300. The p-value = $P(X \geq 4 \mid n=5, p=0.5)$
$= 1 - P(X \leq 3 \mid n=5, p=0.5)$
$= 1 - 0.8125$
$= 0.1875 > 0.05$, so you fail to reject the H_0 at 5% level.

14.2: Single-Population Inferences

If n is large ($n \geq 10$), the normal distribution table can be used. The test statistic is

$$z = \frac{(S - 0.5) - 0.5n}{0.5\sqrt{n}}.$$

and we reject H_0 for the one-tail test if $z \geq z_\alpha$, or reject H_0 for the two-tail test if $z \geq z_{\alpha/2}$.

14.2: Single-Population Inferences

Median time to failure for a band of compact disc players is 5,250 hours. Twenty players from a competitor are tested, with failure times from 5 hours to 6,575 hours. Fourteen of the players exceed 5,250 hours.

Do the competitor's machines perform differently at α = 10% level?

14.2: Single-Population Inferences

$$H_0 : \eta = 5{,}250$$
$$H_a : \eta \neq 5{,}250$$
$$\alpha = .10$$
$$z_{\alpha/2} = z_{0.05} = 1.645$$
$$z = \frac{(S - .5) - 0.5n}{0.5\sqrt{n}} = \frac{13.5 - 10}{0.5\sqrt{20}} = 1.565 < 1.645$$

Do not reject H_0

14.3: Comparing Two Populations: Independent Samples

- Wilcoxon Rank Sum Test

  Used to test whether two independent samples have the same probability distribution

Given random sample 1, $x_1, x_2, \ldots, x_n$ and random sample 2, $y_1, y_2, \ldots, y_m$, from distributions D_1 and D_2, respectively. Sample 1 and sample 2 are independent of each other. Assign ranks 1 to n+m to the combined sample. Denote by T_1 the sum of the ranks sample 1 receives and T_2 the sum of the ranks sample 2 receives.

If there are ties, assign to each the average of the ranks that would have been assigned to them had there been no ties.

For example, if there are tied values from 5th to 8th position, assign 6.5 = (5+6+7+8)/4 to each of the four positions.

14.3: Comparing Two Populations: Independent Samples

Wilcoxon Rank Sum Test

- One-tailed test

  H_0: D_1 and D_2 are identical

  H_a: D_1 is shifted right of D_2

  [**or** H_a: D_1 is shifted left of D_2]

  Test statistic:

  T_1, if $n_1 < n_2$

  T_2, if $n_1 > n_2$

  Either if $n_1 = n_2$

  Rejection region:

  T_1: $T_1 \geq T_U$ [or $T_1 \leq T_L$]

  T_2: $T_2 \leq T_L$ [or $T_2 \geq T_U$]

- Two-tailed test

  H_0: D_1 and D_2 are identical

  H_a: D_1 is shifted either right or left of D_2

  Test statistic:

  T_1, if $n_1 < n_2$

  T_2, if $n_1 > n_2$

  Either if $n_1 = n_2$

  Denote by T the statistic

  Rejection region:

  $T \leq T_L$ or $T \geq T_U$

Reaction Time Under Drug A & B

Drug A (n_1 = 6) Reaction Time	Rank	Drug B (n_2 = 7) Reaction Time	Rank
1.96	4	2.11	6
2.24	7	**2.41**	8.5
1.71	2	2.07	5
2.41	8.5	2.71	11
1.62	1	2.50	10
1.93	3	2.84	12
		2.88	13

$T_1 = 4+7+2+8.5+1+3 = 25.5$

$T_2 = 6+8.5+5+11+10+12+13 = 65.5$

We want to test

H_0: D_A and D_B are identical

H_a: D_A is shifted right of D_B **or** H_a: D_A is shifted left of D_B

Since $6 = n_1 < n_2 = 7$, our test statistic is $T = T_1$. At $\alpha = 0.05$ level, we reject H0 if

$T \le T_L = 28$ or $T \ge T_U = 56$. Now T = 25.5 < 28, we reject H_0 at 5% level to conclude that Drug B seems to increase the reaction time.

14.3: Comparing Two Populations: Independent Samples ($n_1 \ge 10$, $n_2 \ge 10$)

Wilcoxon Rank Sum Test for Large Samples

- One-tailed test
  - H_0: D_1 and D_2 are identical
  - H_a: D_1 is shifted right of D_2
  - [**or** H_a: D_1 is shifted left of D_2]
  - *Rejection region:*
  - $z \ge z_\alpha$ [or $z < -z_\alpha$]
- Two-tailed test
  - H_0: D_1 and D_2 are identical
  - H_a: D_1 is shifted either right or left of D_2
  - *Rejection region:*
  - $|z| \ge z_{\alpha/2}$

Test Statistic: $$z = \frac{T_1 - \frac{n_1(n_1+n_2+1)}{2}}{\sqrt{\frac{n_1 n_2 (n_1+n_2+1)}{12}}}$$