

**PMAT 315**  
**ASSIGNMENTS WINTER 2010**

**Assignment 1.**

1. §1.1, #15. Prove the well-ordering axiom by strong induction. [**Hint:** If  $X$  is a nonempty set of non-negative integers with no smallest member, let  $p_n$  be the statement  $n \notin X$ .] 7 marks
  2. §1.2, #14. Show that  $\gcd(m+n, m) = \gcd(m, n)$ . 7 marks
  3. §1.2, #17. If  $\gcd(m, n) = 1$  and  $\gcd(k, n) = 1$ , show that  $\gcd(mk, n) = 1$ . 6 marks
  4. §1.3, #22(c). In  $\mathbb{Z}_{20}$  find the inverse of  $\overline{11}$  and use it to solve  $\overline{11}x = \overline{16}$ . 6 marks
  5. §1.3, #29(b). Show that  $\bar{a}$  is invertible in  $\mathbb{Z}_n$  if and only if  $\bar{a}\bar{b} = \bar{0}$  implies  $\bar{b} = \bar{0}$ . 7 marks
  6. §1.4, #26. Let  $\gamma$  be any cycle of length  $r$ . If  $\sigma \in S_n$ , show that  $\sigma\gamma\sigma^{-1}$  is also a cycle of length  $r$ .  
More precisely, if  $\gamma = (k_1 k_2 \cdots k_r)$  show that  $\sigma\gamma\sigma^{-1} = (\sigma k_1 \sigma k_2 \cdots \sigma k_r)$ . 7 marks
- Total 40 marks