

ASSIGNMENT 1 SOLUTIONS.

MATH 311

FALL 2010

1. If $\mathbb{R}^n = \text{span}\{X_1, X_2, \dots, X_k\}$, and $A \neq 0$ is $m \times n$, show that $AX_i \neq 0$ for some i . 7 marks
2. Let A denote an $m \times n$ matrix. Show that $\dim[\text{null}(A)] = \dim[\text{null}(AV)]$ for any invertible $n \times n$ matrix V . [Hint: If $AX = 0$ then $AV(V^{-1}X) = 0$.] 7 marks
3. Show that $\{x^2 + 2, 3x - x^3, 4 + x + x^3, x\}$ spans \mathbf{P}_3 . 6 marks
4. Let $U = \text{span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \dots, \mathbf{v}_k\}$ in a vector space V . If $\mathbf{w} = a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + a_3\mathbf{v}_3 + \dots + a_k\mathbf{v}_k$ where $a_2 \neq 0$, show that $U = \text{span}\{\mathbf{v}_1, \mathbf{w}, \mathbf{v}_3, \dots, \mathbf{v}_k\}$. 7 marks
5. Among all independent sets in a vector space V , let $B = \{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k\}$ be an independent set where k is as large as possible. Show that B is a basis of V . 7 marks
6. Find the dimension of the subspace U of \mathbb{R}^5 if $U = \left\{ \left[\begin{array}{c} a - b \\ c + 2b \\ 3b + c \\ c - a \\ 3a - 2b + 5c \end{array} \right] \mid a, b, c \text{ in } \mathbb{R} \right\}$. 6 marks

Total: 40 marks