ASSIGNMENT 6.

MATH 311 **FALL 2010**

1. §9.1 #4(c). Let $T : \mathbf{P}_2 \to \mathbb{R}^2$ be given by $T(a+bx+cx^2) = (a+c, 2b)$. Consider the bases $B = \{1, x, x^2\}$ of $\mathbf{P}_2$ and $D = \{(1,0), (1,-1)\}$ of $\mathbb{R}^2$. Find the matrix of T corresponding to these bases, use it to compute $C_D[T(\mathbf{v})]$, and and use that to compute $T(\mathbf{v})$ where $\mathbf{v} = a + bx + cx^2$. 8 marks

2. §9.1 #14. Let U be an invertible $n \times n$ matrix, and let $D = \{\mathbf{f}_1, \mathbf{f}_2, \cdots, \mathbf{f}_n\}$ where $\mathbf{f}_j$ is column j of U for each j. If B is the standard basis of $\mathbb{R}^n$, show that $M_{BD}(1_{\mathbb{R}^n}) = U$. 8 marks

3. §9.2 #15. Let $B = \{\mathbf{b}_1, \mathbf{b}_2, \cdots, \mathbf{b}_n\}$ be any ordered basis of $\mathbb{R}^n$, written as columns. Define the matrix $Q = [\mathbf{b}_1 \ \ \mathbf{b}_2 \ \ \cdots \ \ \mathbf{b}_n]$ with the $\mathbf{b}_i$ as its columns. Show that $QC_B(\mathbf{v}) = \mathbf{v}$ for all $\mathbf{v} \in \mathbb{R}^n$.

   8 marks

4. §9.3 #16(a). Let $V \xrightarrow{T} W \xrightarrow{S} V$ be linear transformations, and assume that $dim(V)$ and $dim(W)$ are finite. If $ST = 1_V$, show that $V = im(T) \oplus ker(S)$.

   [Hint: If $\mathbf{w} \in W$, first show that $\mathbf{w} - TS(\mathbf{w}) \in ker(S)$.] 8 marks

5. §9.3 #8(a). Define a linear transformation $T : \mathbf{P}_2 \to \mathbf{P}_2$ by

$$T(a + bx + cx^2) = (-a + 2b + c) + (a + 3b + c)x + (a + 4b)x^2.$$

   If $U = span\{1, x + x^2\}$, show that U is T-invariant, find a block upper triangular matrix for T, and use that to compute the characteristic polynomial $c_T(x)$ of T. 8 marks

Total: 40 marks