MATH 271 L02 QUIZ 5 SOLUTIONS April 15, 2010

[9] 1. Let $X = \{1, 2, 3, \ldots, n\}$, where $n \geq 3$ is an integer. Define the relation R on X by: for all $a, b \in X$, aRb if and only if $a + 1 < b$.

(a) Prove that R is not reflexive.

Solution. For example, $1 + 1 \not< 1$, so $1 \not R\, 1$, so R is not reflexive.

(b) Prove that R is not symmetric.

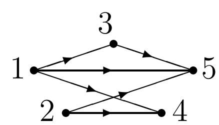
Solution. For example, $1 + 1 < 3$, so $1R3$, but $3 + 1 \not< 1$, so $3 \not R\, 1$. Thus R is not symmetric.

(c) Prove that R is transitive.

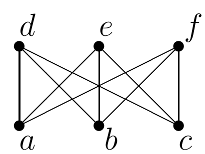
Solution. Assume that $a, b, c \in X$ so that aRb and bRc. This means that $a + 1 < b$ and $b + 1 < c$. Thus $a + 1 < b < b + 1 < c$, so $a + 1 < c$, so aRc. Therefore R is transitive.

(d) Let $n = 5$, so that $X = \{1, 2, 3, 4, 5\}$. Draw the directed graph of the relation R.

Solution.

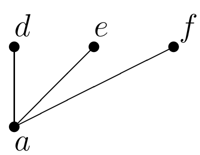


[6] 2. You are given the complete bipartite graph $K_{3,3}$:



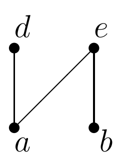
(a) Draw a subgraph of $K_{3,3}$ which has exactly 4 vertices and 3 edges, and which has a vertex of degree 3.

Solution. For example,



(b) Draw a subgraph of $K_{3,3}$ which has exactly 4 vertices and 3 edges, and which has a vertex of degree 2.

Solution. For example,



(c) Find and simplify the number of subgraphs of $K_{3,3}$ that have exactly two vertices.

Solution. Such subgraphs can either have an edge ( ) or have no edge ().

For no edge, we just count the number of 2-element subsets of the vertices $\{a, b, c, d, e, f\}$, and there are $\binom{6}{2} = \frac{6(5)}{2} = 15$ of these.

For an edge, we just count the number of edges, and there are $3(3) = 9$ of these. So any of these with its end vertices would be such a subgraph.

Therefore there are $15 + 9 = 24$ subgraphs with two vertices.

$\overline{[15]}$