

Old Midterm 1:

1. $(-\infty, 2) \cup (5, \infty)$.
2. (a) $\frac{2}{3}$;
(b) 2.
3. (a) 1, (b) $\frac{2}{3} - \frac{a}{3}$, (c) $\frac{3}{2-a}$, (d) -1, (e) 5, (f) 2, (h) Any real number.
4. (a)

$$y' = \frac{(x^2 + 1)(\tan x + x \sec^2 x) - 2x^2 \tan x}{(x^2 + 1)^2};$$

(b)

$$y' = \frac{1}{2} \left((x^2 + 1) \sin^3 x \right)^{-\frac{1}{2}} \left(2x \sin^3 x + 3(x^2 + 1) \sin^2 x \cos x \right).$$

5. (a) $L(x) = 2 - \frac{1}{4}(x - 1)$;
(b) $\sqrt{4.12} = f(0.88) \approx L(0.88) = 2 - \frac{1}{4}(0.88 - 1) = 2.03$;
(c) 0.001.
6. (a) $y' = \frac{3x^2 - y}{2y + x}$, (b) $y - 2 = \frac{5}{3}(x - 2)$.
7. (1, 2).

Old Miterm 2:

1. $(-\infty, 1) \cup [2, 3)$.
2. (a) $f'(x) = -4 \sin 4x \sin^4 x + 4 \cos 4x \sin^3 x \cos x$;
(b) $f'(x) = \frac{(1+x^2)\cos x - 2x \sin x}{(1+x^2)^2}$.
3. (a) $L(x) = \frac{\pi^2}{4} + \pi(x - \frac{\pi}{2})$;
(b) $E(-\frac{\pi}{2}) = |f(-\frac{\pi}{2}) - L(-\frac{\pi}{2})| = \frac{\pi^2}{2}$.
4. $y = x$.
5. (a) 3, (b) 3, (c) the function $f(x)$ is differentiable at $x = 1$ because, $f'_-(1) = 3 = f'_+(1)$, and $f'(1) = 3$, (d) the differentiability of $f(x)$ implies its continuity.

6. $f(x) := x^3 - 15x + 1$ is everywhere continuous, so it has zeros (by the IVT) on any closed interval where it changes sign. Next, $f(0) = 1 > 0$, $f(1) = -13 < 0$, $f(3) = -17 < 0$ and $f(4) = 5 > 0$. Therefore, $f(x)$ has roots in $[0, 1]$ and $[3, 4]$, and the two roots are contained in the interval $[0, 4]$.
7. (a) 1, (b) -3 .