

1. Let

$$\begin{cases} x^2 - 1 & \text{if } x \leq 1 \\ k(x - 1) & \text{if } x > 1. \end{cases}$$

For what value of $k \in \mathbb{R}$ is the function f

(a) continuous at 1? (b) differential at 1?

Answer: (a) f is continuous at 1 for any value of k . (b) $k = 2$ is the only value of k which makes f differentiable at 1.

2. Use the definition of the derivative to find $f'(x)$:

(a) $f(x) = \frac{1}{\sqrt{x-1}}$; (b) $f(x) = \sqrt{2x+8}$.

3. Find the derivative of $f(x)$ if

(a) $f(x) = \frac{\sin(x)}{\cos(x)+\sin(x)}$; (b) $f(x) = \ln(\cos(e^x + 1))$?

4. Let $f(x) = \frac{1}{4x+5}$.

(a) Find the local linear approximation of $f(x)$ at $x = 0$.

(b) Use the linear approximation of $f(x)$ to estimate the value of $\frac{1}{5.4}$.

Answer: (a) $L(x) = f(0) + f'(0)(x - 0)$, where $f(0) = 1/5 = 0.2$ and $f'(0) = -4/25 = -0.16$, i.e., $L(x) = 0.2 - 0.16x$. (b) $\frac{1}{5.4} = \frac{1}{4(0.1)+5} = f(0.1) \approx L(0.1) = 0.2 - 0.016 = 0.184$.