

THE UNIVERSITY OF CALGARY
DEPARTMENT OF MATHEMATICS AND STATISTICS
FINAL EXAMINATION
STATISTICS 213 SAMPLE EXAM

Time: 2 hours

I.D. NUMBER	SURNAME	OTHER NAMES

STUDENT IDENTIFICATION

Each candidate must sign the Seating List confirming presence at the examination. All candidates for final examinations are required to place their University of Calgary student I.D. cards on their desks for the duration of the examination. (Students writing mid-term tests can also be asked to provide identity proof.) Students without an I.D. card who can produce an **acceptable** alternative I.D., e.g., one with a printed name and photograph, are allowed to write the examination.

A student without acceptable I.D. will be required to complete an Identification Form. The form indicates that there is no guarantee that the examination paper will be graded if any discrepancies in identification are discovered after verification with the student's file. A student who refuses to produce identification or who refuses to complete and sign the Identification Form is not permitted to write the examination.

EXAMINATION RULES

1. Students late in arriving will not normally be admitted after one-half hour of the examination time has passed.
2. No candidate will be permitted to leave the examination room until one-half hour has elapsed after the opening of the examination, nor during the last 15 minutes of the examination. All candidates remaining during the last 15 minutes of the examination period must remain at their desks until their papers have been collected by an invigilator.
3. All enquiries and requests must be addressed to supervisors only.
4. Candidates are strictly cautioned against:
 - (a) speaking to other candidates or communicating with them under any circumstances whatsoever;
 - (b) bringing into the examination room any textbook, notebook or memoranda not authorized by the examiner;
 - (c) making use of calculators and/or portable computing machines not authorized by the instructor;
 - (d) leaving answer papers exposed to view;
 - (e) attempting to read other students' examination papers.

The penalty for violation of these rules is suspension or expulsion or such other penalty as may be determined.

5. Candidates are requested to write solutions in the space provided. If there is insufficient room you may write on the back of the page, but indicate clearly you are doing so.
6. Discarded matter is to be struck out and not removed by mutilation of the examination answer book.
7. Candidates are cautioned against writing in their answer books any matter extraneous to the actual answering of the question set.
8. The candidate is to write his/her name on each answer book as directed and is to number each book.
9. A candidate must report to a supervisor before leaving the examination room.
10. Answer books must be handed to the supervisor-in-charge promptly when the signal is given. Failure to comply with this regulation will be cause for rejection of an answer paper.
11. If a student becomes ill or receives word of domestic affliction during the course of an examination, he/she should report at once to the Supervisor, hand in the unfinished paper and request that it be cancelled. Thereafter, if illness is the cause, the student must go directly to University Health Services so that any subsequent application for a deferred examination may be supported by a medical certificate. An application for Deferred Final Examinations must be submitted to the Registrar by the date specified in the University Calendar.
Should a student write an examination, hand in the paper for marking, and later report extenuating circumstances to support a request for cancellation of the paper and for another examination, such request will be denied.
12. CELL PHONE USE DURING EXAMINATIONS IS STRICTLY PROHIBITED.



NOTE: A nongraphing calculator *is* allowed.

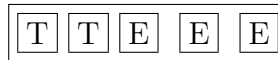
Formula sheet and Normal and Poisson Distribution Tables are attached.

PART A. **Multiple Choice.** Circle the letter corresponding to the correct answer.

- Suppose that the random variable X has mean $\mu = 5$ and standard deviation $\sigma = 8$. The mean and standard deviation of $Y = 2X - 4$ is:
 - $\mu_Y = 6, \sigma_Y = 16$
 - $\mu_Y = 6, \sigma_Y = 4$
 - $\mu_Y = 10, \sigma_Y = 16$
 - $\mu_Y = 6, \sigma_Y = 32$
 - none of the above
- 225 samples of water specimens are taken from a lake at random. The sample mean alkalinity is 50.0 mg per litre and the sample standard deviation is 4.0 mg per litre. According to Chebyshev's rule (inequality) at least how many samples have alkalinity level between 44.0 and 56.0?
 - 75
 - 111
 - 125
 - 100
 - none of the above
- A manufacturer has stockpiled 25 units of a particular product in a warehouse. Unknown to the manufacturer, 5 of them are defective. A clerk samples 14 units at random without replacement and examines them. The probability that exactly 3 of the sampled units are defective equals:
 - $\frac{e^{-5}5^3}{3!}$
 - $\frac{\binom{5}{3}\binom{20}{11}}{\binom{25}{14}}$
 - $\binom{14}{3} \left(\frac{5}{25}\right)^3 \left(\frac{20}{25}\right)^{11}$
 - $\left(\frac{5}{25}\right)^3$
 - none of the above
- Events A and B are independent and $P[A]$ and $P[B]$ are both greater than zero. Which statement below is NOT correct.
 - $P[A \text{ or } B] = P[A] + P[B]$
 - $P[B | A] = P(B)$
 - $P[A \text{ and } B] = P(A)P(B)$
 - $P[A | B] = P(A)$

5. According to an opinion poll, 54% of US men, and 33% of US women believe in aliens. Of US adults 48% percent are men, and 52% percent are women. What percentage of US adults believe in aliens?
- (a) 0.25
 - (b) 0.65
 - (c) 0.37
 - (d) 0.43
 - (e) none of the above
6. During the first week of December, customers call to order from the catalogue of a major department store at a rate of 6 per hour. Assume that the number of customers calling is a random variable with a Poisson distribution. The probability that there would be at least 5 calls between 11:00 and 11:30 on December 6th is:
- (a) .715
 - (b) .185
 - (c) .285
 - (d) .815
 - (e) None of the above.
7. The probability that a particular computer component has a defect is .002. A random sample of such 1500 components is taken. The probability that there would be more than 3 components defective in this sample, by using the Poisson approximation, is:
- (a) .125
 - (b) .353
 - (c) .692
 - (d) .500
 - (e) none of the above
8. We wish to select three numbers from 1, 2, 3, 4, 5 at random with replacement: Five identically sized cards numbered 1 to 5 are placed in a hat. Three cards are selected at random from the hat one at a time. Each time a card is selected its number is recorded and put back into the bag. The cards are then mixed before the next selection. The probability that two or more of the selected numbers are the same is:
- (a) $\frac{5}{25}$
 - (b) $\frac{5}{125}$
 - (c) $\frac{12}{25}$
 - (d) $\frac{13}{25}$
 - (e) 1

9. Two cards are chosen at random without replacement from the following box:



- Let A be the event that the first card chosen is a “T” and let B be the event that the second card chosen is a “T”. Which of the following is true:
- (a) A and B are both independent and mutually exclusive.
 - (b) A and B are independent but not mutually exclusive.
 - (c) A and B are mutually exclusive but not independent.
 - (d) A and B are neither mutually exclusive nor independent.
10. A consumers’ group wishes to test 5 kg. bags of sugar of a certain company to see if they underfill them. At level $\alpha = .02$, they wish to prove that the average amount of sugar for all bags produced by this company is less than 5 kg., based on a random sample of 20 bags. Their statistical hypotheses are:
- (a) The null hypothesis is that the sample average content is equal to 5 kg.; the alternative hypothesis is that the sample average content is less than 5 kg.
 - (b) The null hypothesis is that the population average content is less than 5 kg.; the alternative hypothesis is that the population average content is equal to 5 kg.
 - (c) The null hypothesis is that the sample average content is greater than 5 kg.; the alternative hypothesis is that the sample average content is equal to 5 kg.
 - (d) The null hypothesis is that the population average content is equal to 5 kg.; the alternative hypothesis is that the population average content is less than 5 kg.
 - (e) None of the above.

11. The proportion of students who own a cell phone on college campuses across the country has increased tremendously over the past few years. It is estimated that approximately 90% of students now own a cell phone. Ten students are to be selected at random from a large university. Assume that the proportion of students who own a cell phone at this university is the same as nationwide. Let X = the number of students in the sample of 10 who own a cell phone. What is the standard deviation of X ?
- (a) .95
 - (b) .9
 - (c) 3
 - (d) 1.05
 - (e) 9
12. A box contains 5 red, 6 blue, and 4 green marbles. 7 marbles are randomly picked from the box, without replacement. Find the probability that the 7-marble sample consists of exactly 3 red, 1 green, and 3 blue marbles.
- (a) .012
 - (b) .124
 - (c) .231
 - (d) .349
 - (e) None of the above.

PART B. **Problems.** Answer in the space provided. If there is not sufficient room, use the back of the page, but INDICATE that you are doing so.

1. In the city of Red Deer, a random sample of 700 adults was taken. It was found that 224 of the adults in the sample were smokers.

(a) Find a 98% confidence interval for the population proportion of Red Deer adults who are smokers.

(b) Without performing *all* the calculations, would a 95% confidence interval be wider or narrower? Why?

2. A new type of light bulb is tested. A random sample of 49 bulbs is taken. Each bulb is used continually until it burns out. The average lifetime of the sample was 477.29 hours and the sample standard deviation was 63.70 hours. Determine if there is enough evidence to conclude that the population mean lifetime for all such light bulbs is greater than 460 hours.

(Perform a test of hypotheses using level of significance $\alpha = .05$. State the null hypothesis and the alternative hypothesis, your test statistic, the decision rule, and whether the null hypothesis should be rejected in favour of the alternative.)

3. It is known that 20% of university students have missed class during the semester because of drinking. A random sample of 50 students is taken. Let X equal the number of them who have missed class because of drinking.

(a) Find $P[8 \leq X \leq 13]$, using the normal approximation. Use the continuity correction.

(b) Find $P[X = 9]$, using the normal approximation.

End of Examination

FORMULA SHEET

$$P(\bar{x} - ks < X < \bar{x} + ks) \geq 1 - \frac{1}{k^2},$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B),$$

$$P(A) = \sum P(A | B_i) P(B_i), \quad P(B_i | A) = \frac{P(A | B_i) P(B_i)}{\sum P(A | B_j) P(B_j)},$$

$$P(X = k) = \binom{n}{k} p^k q^{n-k}, \quad \mu = np, \quad \sigma^2 = npq,$$

$$P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}, \quad \mu = \lambda, \quad \sigma^2 = \lambda,$$

$$P(X = k) = \frac{\binom{M}{k} \binom{N-M}{n-k}}{\binom{N}{n}},$$

$$\sigma^2(aX + bY) = a^2\sigma^2(X) + b^2\sigma^2(Y) + 2abCov(X, Y),$$

$$P\left(\bar{x} - z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} < \mu < \bar{x} + z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}\right) = 1 - \alpha,$$

$$P\left(\bar{x} - t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}\right) = 1 - \alpha,$$

$$P\left(\hat{p} - z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}}\right) = 1 - \alpha,$$

$$z_0 = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}, \quad z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0q_0}{n}}}$$

$$\beta_0 + \beta_1 \bar{x} = \bar{y}$$

$$n\beta_0 \bar{x} + \beta_1 \sum x_i^2 = \sum x_i y_i$$

$$r = \frac{SS_{xy}}{\sqrt{SS_{xx}SS_{yy}}}.$$

ANSWERS

PART A

1. a)
2. c)
3. b)
4. a)
5. d)
6. b)
7. b)
8. d)
9. d)
10. d)
11. a)
12. b)

PART B

1. a) $CI=(0.28,0.36)$ b) Narrower

2. $z_0 = 1.9$, H_0 rejected

3. a) 0.7031 b) 0.1305