

Pmat 421
Practice Midterm 1

1. For $f(z) = \frac{z}{3\bar{z} - 2}$ find the domain of definition and the functions u and v such that $f(z) = u(x, y) + iv(x, y)$ for $z = x + iy$. Is it onto C ? Explain.
 2. Find all z such that $e^{iz} + 3 = 0$.
 3. Prove that the mapping $w = e^z$ is one-to-one on $S = \{|z| < \pi\}$.
Is it one-to-one on $\bar{S} = \{|z| \leq \pi\}$?
 4. For all $z \neq 0$ define the branch of $\arg z \in [0, 2\pi)$ in terms of x and y if $z = x + iy$.
 5. Prove that $|z| \geq \frac{1}{\sqrt{2}} (|\operatorname{Re} z| + |\operatorname{Im} z|)$.
 6. Sketch the set $S = \{0\} \cup \left\{ z ; \left| \frac{\operatorname{Re} z}{\operatorname{Im} z} \right| \geq 1 \right\}$; find the boundary ∂S .
Is S open, closed, connected, simply connected, bounded? Explain
 7. Solve $(e^z + 1)^2 = e^z$
 8. Find $\lim_{z \rightarrow i} \frac{[\operatorname{Im}(z - i)]^2}{z - i}$
 9. Compare two functions $f(z) = |z|^2$ (complex valued of complex variable) and $g(x) = |x|^2$ (real valued of real variable).
 - (a) Where is g continuous, and where is differentiable?
 - (b) Where is f continuous and where is differentiable?
 - (c) Where is f analytic?
- EXPLAIN!
10. Show that $u(x, y) = x^2 - y^2 + x - y$ is harmonic everywhere, find a harmonic conjugate $v(x, y)$ and then find $f(z) = u + iv$ in terms of z .
 11. Find and describe the image of the set S under the mapping $w = (3 + 2i)z + 2 - i$
 - (a) if S is the circle with the center i and radius $\sqrt{2}$.
 - (b) if S is the line $y = x$ in the z -plane.
 12. Find the limit $\lim_{z \rightarrow \infty} \frac{\operatorname{Im} z}{z}$ if it exists.