

**Pmat 421**  
**Assignment # 3 due by Monday March 10 , 4pm.**

Each questions is worth 5 points.

1. Find all solutions of  $\sin z = i$  in the form of  $a + ib, a, b$  real.  
Which one is the principal value?
2. Find all values of (a)  $i^{-1-i}$   
(b)  $(-1 - i)^i$  in the form of  $a + ib, a, b$  real.
3. Solve for  $z$ :  $\cos z = i \sin z$  if possible.
4. Find the domain of definition and derivative of  
(a)  $f(z) = \tan^3 z$ ;  
(b)  $f(z) = \cos 2z + \sin \frac{1}{z}$ .
5. For  $f(z) = \text{Log}(3z - i)$  find the domain  $D$  where the function is analytic and then  $f'(z)$  in  $D$ . ( $\text{Log}$  is the principal branch.)
6. Solve for all  $z$   $\text{Log}(z^2 - 1) = i\frac{\pi}{2}$ .
7. Evaluate the following limit if it exists:  $\lim_{z \rightarrow \infty} e^{-z}$
8. Evaluate  $\int_0^1 (1 + 2it)^3 dt$  in the form of  $a + ib, a, b$  real.
9. Evaluate  $\int_0^2 \frac{t}{(t^2 + i)^2} dt$ . in the form of  $a + ib, a, b$  real.
10. Evaluate  $\int_{-2}^0 (1 + i) \cos it dt$  in the form of  $a + ib, a, b$  real.  
BONUS QUESTION for 10 points:
11. Find the conditions on a function  $f$  which is analytic in a domain  $D$  such that  $\text{Re}(f'(z)) = 0$  for all  $z \in D$ .