

**Pmat 421      Winter 08**  
**Assignment # 1      due by Monday, Jan 28 by 4 pm**

Each questions is worth 5 points.

1. Express in the form  $a + ib$ , with  $a, b$  real:  $\left(\frac{i}{1-i} + \frac{1-i}{i}\right)^3$ .
2. Find all  $z$  (i.e. conditions /restriction on  $z$ ) for which  $(\bar{z})^2 = -|z|^2$ .
3. Sketch/Describe the set (a)  $|z + i| \leq 2$ ;      (b)  $z^2 + (\bar{z})^2 = 2$ .
4. Express in the form  $a + ib$ , with  $a, b$  real:  $(-1 - i)^8 (1 + i\sqrt{3})^4$ , use polar form first.
5. For  $z = \left(\frac{1+i}{1-i}\right)^3$  find  
(a) (principal branch)  $\text{Arg } z$ ; then (b) find both roots  $\sqrt{z}$ .
6. Find all  $z \neq 0$  for which  $\text{Arg} \frac{1}{z} = -\text{Arg}(z)$ .  
Explain why it is NOT always true.
7. Show that  $|z + w| \leq |z| + |w|$  if and only if  $\arg z = \arg w$ .  
You may use geometry.
8. Find all fourth roots of  $-1$  in the form  $a + ib$ , with  $a, b$  real.  
Sketch them on the unit circle.
9. Use De Moivre's Theorem to express  $\cos(4\theta)$  in terms of  $\cos \theta$  and  $\sin \theta$ .
10. Sketch the set  $\{\text{Im}(z^2) > 1\}$ . Is the set open, closed, bounded, connected?  
EXPLAIN.