The University of Calgary
Department of Mathematics and Statistics
MATH 353-02
Quiz #5T(3pm)

Winter 2008

Name:______________________________**I.D.#:**__________________

1. Evaluate $\iint\limits_S y^2 z\ dS$ and S is the part of the cone $\qquad z = \sqrt{y^2 + x^2}$

   between two planes $z = 1$ and $z = 2$. [3]

2. Find $\iint\limits_S \mathbf{F} \bullet \mathbf{dS}$ where $\mathbf{F}(x, y, z) = (xz, 1 + 2(y - 4)z, 2x^2 + 3y^2 + z^2)$ and S is the part of

   the vertical sheet $y = 4 - x^2, (z$ any) below the plane $5x + 3y + z = 15$ in the first octant

   oriented in the direction of positive x-axis. [4]

3. For $\mathbf{F}(x, y, z) = (\sin(x^2 z + y^2), \ln(\dfrac{x^2 + y^2}{z}), xz)$ find div $\mathbf{F}$ and curl $\mathbf{F}$

   for $z > 0, (x, y) \neq (0, 0)$. [3]

SOLUTION

For 1)

S is given by $z = \sqrt{x^2 + y^2}$ for $(x, y) \in D = \{1 \leq x^2 + y^2 \leq 4\}$

$$\mathbf{n} = \left(\frac{x}{\sqrt{x^2 + y^2}}, \frac{y}{\sqrt{x^2 + y^2}}, -1\right) \qquad \|\mathbf{n}\| = \sqrt{\frac{x^2 + y^2}{x^2 + y^2} + 1} = \sqrt{2}$$

f on S

$$\iint\limits_S y^2 z\ dS = \sqrt{2} \iint\limits_D y^2 \sqrt{x^2 + y^2}\ dxdy = \text{(by polar.coord.)}$$

$$= \sqrt{2} \int_0^{2\pi} \int_1^2 r^2 \sin^2\theta\ r^2 dr d\theta = \sqrt{2} \int_0^{2\pi} \frac{1 - \cos 2\theta}{2}\ d\theta \int_1^2 r^4 dr = \sqrt{2}\pi \left(\frac{2^5}{5} - \frac{1}{5}\right) = \frac{31}{5}\sqrt{2}\pi.$$

For 2)

below $\rightarrow 0 \leq z \leq 15 - 5x - 3y$

S is a part of vertical surface $\rightarrow$ parametrization

we can choose $\qquad x = u \qquad y = 4 - u^2 \qquad z = v$

$\mathbf{r}(u, v) = (u, 4 - u^2, v)$ for $u \in [0, 2]$and $0 \leq v \leq 3 - 5u + 3u^2$

$$\frac{\partial \mathbf{r}(u, v)}{\partial u} = (1, -2u, 0) \qquad \frac{\partial \mathbf{r}(u, v)}{\partial v} = (0, 0, 1)$$

cross product $\quad \mathbf{n} = +(2u, 1, 0)$ - positive x-axis

$\mathbf{F}$ on $S \quad \mathbf{F}(x, y, z) = (uv, 1 - 2u^2v, ...)$and $\mathbf{F} \bullet \mathbf{n} = 1$

$$\iint_S \mathbf{F} \bullet \mathbf{dS} = \iint_D \mathbf{F} \bullet \mathbf{n} \; dudv \text{ where } D\left\{(u, v)\,; u \in [0, 2]\,, 0 \leq v \leq 3 - 5u + 3u^2\right\}$$

$$= \int_0^2 \Big(\int_0^{3-5u+3u^2} dv\Big) du = \int_0^2 (3 - 5u + 3u^2) du = 6 - \frac{5}{2} \cdot 4 + 8 = 4.$$

For 3)

$$\operatorname{div} \mathbf{F} = \partial_x \left(\sin(x^2z + y^2)\right) + \partial_y \ln\Big(\frac{x^2 + y^2}{z}\Big) + \partial_z(xz) = 2xz \cos\left(x^2z + y^2\right) + \frac{2y}{x^2 + y^2} + x$$

$$\mathbf{curl}(\mathbf{F}) = \begin{vmatrix} + & - & + \\ \partial_x & \partial_y & \partial_z \\ \sin(x^2z + y^2) & \ln\Big(\dfrac{x^2 + y^2}{z}\Big) & xz \end{vmatrix} =$$

$$= \left(-\frac{1}{z}, -z + x^2 \cos(x^2z + y^2), \frac{2x}{x^2 + y^2} - 2y \cos(x^2z + y^2) \right)$$