Faculty of Science
Department of Mathematics & Statistics

FINAL EXAMINATION - MATH 221 L18 - FALL 2004

December 17, 2004
Time: 3 hours
NOTE: Calculators allowed.

I.D. NUMBER	SURNAME	OTHER NAMES

STUDENT IDENTIFICATION

Each candidate must sign the Seating List confirming presence at the examination. All candidates for final examinations are required to place their University of Calgary student I.D. cards on their desks for the duration of the examination. (Students writing mid-term tests can also be asked to provide identity proof.) Students without an I.D. card who can produce an **acceptable** alternative I.D., e.g., one with a printed name and photograph, are allowed to write the examination.

A student without acceptable I.D. will be required to complete an Identification Form. The form indicates that there is no guarantee that the examination paper will be graded if any discrepancies in identification are discovered after verification with the student's file. A student who refuses to produce identification or who refuses to complete and sign the Identification Form is not permitted to write the examination.

EXAMINATION RULES

1. Students late in arriving will not normally be admitted after one-half hour of the examination time has passed.
2. No candidate will be permitted to leave the examination room until one-half hour has elapsed after the opening of the examination, nor during the last 15 minutes of the examination. All candidates remaining during the last 15 minutes of the examination period must remain at their desks until their papers have been collected by an invigilator.
3. All enquiries and requests must be addressed to supervisors only.
4. Candidates are strictly cautioned against:
   (a) speaking to other candidates or communicating with them under any circumstances whatsoever;
   (b) bringing into the examination room any textbook, notebook or memoranda not authorized by the examiner;
   (c) making use of calculators and/or portable computing machines not authorized by the instructor;
   (d) leaving answer papers exposed to view;
   (e) attempting to read other students' examination papers.

   The penalty for violation of these rules is suspension or expulsion or such other penalty as may be determined.
5. Candidates are requested to write on both sides of the page, unless the examiner has asked that the left half page be reserved for rough drafts or calculations.
6. Discarded matter is to be struck out and not removed by mutilation of the examination answer book.
7. Candidates are cautioned against writing in their answer books any matter extraneous to the actual answering of the question set.
8. The candidate is to write his/her name on each answer book as directed and is to number each book.
9. A candidate must report to a supervisor before leaving the examination room.
10. Answer books must be handed to the supervisor-in-charge promptly when the signal is given. Failure to comply with this regulation will be cause for rejection of an answer paper.
11. If a student becomes ill or receives word of domestic affliction during the course of an examination, he/she should report at once to the Supervisor, hand in the unfinished paper and request that it be cancelled. Thereafter, if illness is the cause, the student must go directly to University Health Services so that any subsequent application for a deferred examination may be supported by a medical certificate. An application for Deferred Final Examinations must be submitted to the Registrar by the date specified in the University Calendar. **Should a student write an examination, hand in the paper for marking, and later report extenuating circumstances to support a request for cancellation of the paper and for another examination, such request will be denied.**
12. SMOKING DURING EXAMINATIONS IS STRICTLY PROHIBITED.

QUESTION	#1	#2	#3	#4	#5	#6	TOTAL
MARKS	/6	/8	/6	/8	/6	/6	/40

Question 1 (6 points)

Consider the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 1 & 0 & a \end{bmatrix}$.

[1] **a)** Find all values of a such the system of linear equations $AX = B$ has a unique solution for some B.

[1] **b)** Find all values of a such the system of linear equations $AX = B$ has a unique solution for all B.

[1] **c)** Find all values of a such the system of linear equations $AX = B$ has infinitely many solutions for some B.

[1] **d)** For these values of a from part c), describe all B such that $AX = B$ has infinitely many solutions.

[1] **e)** Find all values of a such the system of linear equations $AX = B$ has no solutions for some B.

[1] **f)** For these values of a from part a), describe all B such that $AX = B$ has no solutions.

Question 2 (8 points)

Consider the matrix $A = \begin{bmatrix} 1/2 & 1 \\ 1/2 & 0 \end{bmatrix}$.

It turns out that A is diagonalizable, and in fact

$$\begin{bmatrix} 1/3 & 1/3 \\ -1/3 & 2/3 \end{bmatrix} \begin{bmatrix} 1/2 & 1 \\ 1/2 & 0 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1/2 \end{bmatrix}$$

which we can write as $P^{-1}AP = D$.

[2] a) Express the matrix power A^k in terms of only the matrices P, P^{-1} and D.

[3] b) Use your expression in part a) to compute A^5.

[3] c) What can you say about A^k for large values of k?

Question 3 (6 points)

Consider the matrix $A = \begin{bmatrix} 2 & 4 \\ -2 & -2 \end{bmatrix}$

[2] **a)** Show that $z = 2i$ is an eigenvalue of A.

[1] **b)** Does A have any other eigenvalues?

[3] **b)** Find all eigenvectors corresponding to the eigenvalue $z = 2i$.

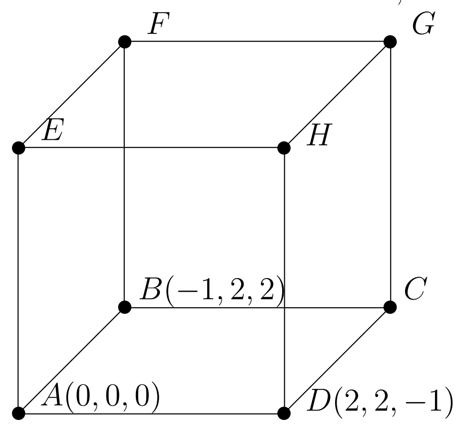
Question 4 (8 points)
Consider the line L containing the points $A(1,1,1)$ and $B(2,3,0)$.

[4] **a)** Find the shortest distance between the point $P(1,-1,3)$ and the line L.

[2] **b)** Find the point Q on L closest to P.

[2] **c)** Draw as accurate a diagram as you can of the line L, including points A, B, P and Q, showing where each point is located in relation to the others.

Question 5 (6 points)
Consider the following cube with vertices as indicated, and where three of them are known:



[2] **a)** Find the coordinates of the point E.

[2] **b)** Find the coordinates of all other points.

[1] **c)** Find the volume of the cube.

[1] **d)** Write an equation for the plane P containing the points A, B and D.

Question 6 (6 points)
Consider the linear transformation $T : \mathbb{R}^2 \Rightarrow \mathbb{R}^2$ such that:

$$T\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2\sqrt{2} \end{bmatrix} \text{ and } T\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \sqrt{2} \\ \sqrt{2} \end{bmatrix}$$

[2] **a)** Express T as a matrix transformation.

[1] **b)** Draw the image of the unit square under T.

[1] **c)** Do you recognize T as a familiar transformation?

[2] **d)** Find the inverse transformation T^{-1}.

End of Examination