MATH 271 L01 QUIZ 5 SOLUTIONS

1. Let $A = \{1,2,3\}$ and $B = \{1,2,3,4\}$.

[2] (a) Give an example of a one-to-one function $f : A \to B$.

Solution: One example is f defined by $f(1) = 1$, $f(2) = 2$, $f(3) = 3$.

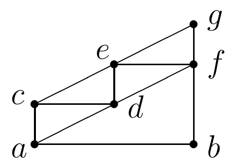
[2] (b) Give an example of an onto function $g : B \to A$.

Solution: One example is g defined by $g(1) = 1$, $g(2) = 2$, $g(3) = 3$, $g(4) = 3$.

[2] (c) Find the composition function $g \circ f : A \to A$, where f and g are the functions you gave in parts (a) and (b).

Solution: We get $(g \circ f)(1) = g(f(1)) = g(1) = 1$, $(g \circ f)(2) = g(f(2)) = g(2) = 2$, $(g \circ f)(3) = g(f(3)) = g(3) = 3$.

2. Let G be the graph



[3] (a) Does G have a Eulerian circuit? Explain.

Solution: No, because G contains vertices of odd degree, namely a and c, both of which have degree 3.

[2] (b) Does G have a Hamiltonian circuit? Explain.

Solution: Yes, for example $acdegfba$ is a Hamiltonian circuit (where we have listed the vertices and not the edges).

3. Define the relation R on $\mathbb{Z}$ by: for all $a, b \in \mathbb{Z}$, aRb if and only if $a + 2b \neq 5$.

[2] (a) Give an example of integers a and b so that a is **not** related to b by R. Explain why your answer is correct.

Solution: One example is $a = 5$, $b = 0$. Then a is not related to b by R, because $a + 2b = 5 + 0 = 5$.

[2] (b) Is R reflexive? Explain.

Solution: Yes, R is reflexive. Here is a proof. Let $A \in \mathbb{Z}$ be arbitrary. Then $a + 2a = 3a$ which is never equal to 5 because 3 does not divide into 5. Thus a is always related to itself, so R is reflexive.

$\overline{[15]}$