

MATH 271 ASSIGNMENT 3

Due 4:00 PM Friday, November 6, 2009. Hand your assignment to me in class or in the lab, or in my office (MS566, under the door if I'm not there). Assignments must be understandable to the marker (i.e., logically correct as well as legible), and of course must be done by the student in her/his own words. Answer **all** questions; but only one question per assignment will be marked for credit.

Marked assignments will be handed back during your scheduled lab, or in class.

1. Prove or disprove each of the following statements. Proofs should use the “element” methods given in Section 5.2. [Note: $\mathcal{P}(X)$ denotes the power set of the set X .]

- (a) For all sets A, B, C , $A \times (B - C) \subseteq (A \times B) - (A \times C)$.
- (b) For all sets A, B, C , $(A \times B) - (A \times C) \subseteq A \times (B - C)$.
- (c) For all sets A, B, C , $A \times (B - C) = (A \times B) - (A \times C)$.
- (d) For all sets A and B , $\mathcal{P}(A) - \mathcal{P}(B) \subseteq \mathcal{P}(A - B)$.
- (e) For all sets A and B , $\mathcal{P}(A - B) \subseteq \mathcal{P}(A) - \mathcal{P}(B)$.
- (f) For all sets A and B , $\mathcal{P}(A - B) = \mathcal{P}(A) - \mathcal{P}(B)$.

2. (a) Prove that

$$n - \left\lceil \frac{n-1}{2} \right\rceil = \left\lfloor \frac{n}{2} \right\rfloor \quad \text{for all positive integers } n.$$

Here $\lceil x \rceil$ denotes the *ceiling* of the number x , as defined in §3.5. [Hint: do the cases n odd and n even separately.]

- (b) The sequence A_0, A_1, A_2, \dots of sets is defined as follows:

$$A_0 = \emptyset, \quad \text{and } A_n = \{1, 2, \dots, n\} - A_{n-1} \text{ for all integers } n \geq 1.$$

Find A_1 , A_2 and A_3 .

- (c) For the sets A_n defined in part (b), prove **by induction on n** that $N(A_n) = \lceil n/2 \rceil$ for every integer $n \geq 0$. [Hint: Theorem 6.3.2 on page 322. $N(X)$ denotes the number of elements in the set X .]

3. A licence plate consists of any three letters (from the usual 26-letter alphabet) followed by any three digits. Find the number of licence plates with each of the following properties. You need not simplify your answers.

- (a) They contain exactly two different symbols.
- (b) They contain exactly three different symbols.
- (c) They contain at least two 8's, but no 4, and the letters HAL in some order.
- (d) They use three different letters in alphabetical order and three different digits in increasing order. [Hint: start by choosing the three letters.]