MATH 271 ASSIGNMENT 2

Due 4:00 PM Friday, October 23, 2009. Hand your assignment to me in class or in the lab, or in my office (MS566, under the door if I'm not there). Assignments must be understandable to the marker (i.e., logically correct as well as legible), and of course must be done by the student in her/his own words. Answer **all** questions; but only one question per assignment will be marked for credit.

Marked assignments will be handed back during your scheduled lab, or in class.

1. Let
$$S_n = \frac{1}{n(n+1)} + \frac{1}{(n+1)(n+2)} + \frac{1}{(n+2)(n+3)} + \cdots + \frac{1}{(2n-1)(2n)} .$$
   (a) Find and simplify S_1, S_2 and S_3.
   (b) Use part (a) (and more data if you need it) to guess a simple formula for S_n for any positive integer n.
   (c) **Use mathematical induction** (or well ordering) to prove that your guess in part (b) is true for all positive integers n.

2. The sequence $b_0, b_1, b_2, \ldots$ is defined by: $b_0 = 1$, $b_1 = 2$ and $b_n = 3b_{n-1} + b_{n-2}$ for all integers $n \geq 2$.
   (a) Calculate b_2, b_3 and b_4.
   (b) **Use mathematical induction** (or well ordering) to prove that $\gcd(b_{n+1}, b_n) = 1$ for all integers $n \geq 0$. [You may use Lemma 3.8.2 on page 193.]
   (c) **Use strong induction** (or well ordering) to prove that $b_n \leq 4^n$ for all integers $n \geq 0$.

3. You are given the following "while" loop:

   [*Pre-condition*: m is a nonnegative integer, $a = 0$, $b = 1$, $i = 0$.]

   while $(i \neq m)$
      1. $a := 2b - a$
      2. $b := 3a - 2b$
      3. $i := i + 1$

   end while

   [*Post-condition*: $b - a = 2^m$.]

   Loop invariant $I(n)$ is: $i = n, \quad a = 2^{n+1} - 2, \quad b = 2^{n+1} + 2^n - 2$.

   (a) Prove the correctness of this loop with respect to the pre- and post-conditions.
   (b) Suppose the "while" loop is as above, with the same pre-condition, except that statement 2 is replaced by: $b := 3a - 2b - 1$. Run through this new loop a few times to get data. Then find a post-condition that gives the final value of $b - a$, and an appropriate loop invariant, and prove the correctness of this new loop.