MATHEMATICS 271 MIDTERM COVER PAGE March 13, 2008

NAME__________________________________ Lecture Section/Professor____________

NAME________________________________ID______________Section________

MATHEMATICS 271 MIDTERM March 13, 2008

SHOW ALL WORK. Marks for each problem are to the left of the problem number.
NO CALCULATORS PLEASE.

[6] 1. **Use the Euclidean algorithm** to find $\gcd(72, 17)$. Then use your work to write $\gcd(72, 17)$ in the form $72a + 17b$ where a and b are integers.

[6] 2. Let $\mathcal{S}$ be the statement:

for all sets A and B, if $A \cup B = \{1, 2\}$ then $\{1, 2\} \in \mathcal{P}(A) \cup \mathcal{P}(B)$.

(Here $\mathcal{P}(X)$ denotes the power set of the set X.)

(a) Write (as simply as possible) the *negation* of statement $\mathcal{S}$.

(b) *Disprove* statement $\mathcal{S}$.

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[11] 3. Let $\mathcal{S}$ be the statement:

$$\text{for all integers } n, \text{ if } 6 \mid n \text{ then } 9 \mid (n^2 + 3n).$$

(a) Is $\mathcal{S}$ true? Give a proof or disproof.

(b) Write out (as simply as possible) the *contrapositive* of statement $\mathcal{S}$. Is it true or false? Explain.

(c) Write out (as simply as possible) the *converse* of statement $\mathcal{S}$. Is it true or false? Explain.

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[6] 4. Of the following two statements, one is true and one is false. Prove the true statement. Disprove the false statement by writing out its negation and proving that. ($\mathbb{Z}$ denotes the set of all integers.)

(a) $\forall A \subseteq \mathbb{Z}\ \exists B \subseteq \mathbb{Z}$ so that $1 \in B - A$.

(b) $\forall A \subseteq \mathbb{Z}\ \exists B \subseteq \mathbb{Z}$ so that $1 \notin B - A$.

[5] 5. You are given that A and B are arbitrary subsets of the set $\mathbb{Z}$ of all integers such that $A \cap B = \{1\}$.

(a) Find an element of $A \times B$. Explain.

(b) Find an element of the complement $(A \times B)^c$. (Here assume the universal set is $\mathbb{Z} \times \mathbb{Z}$.) Explain.

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[6] 6. Prove **using mathematical induction** (or well ordering) that $2^n + 2n \leq 3^n$ for all integers $n \geq 2$.

$\overline{[40]}$