MATHEMATICS 271 MIDTERM March 8, 2007

[6] 1. **Use the Euclidean algorithm** to find $\gcd(100, 43)$. Then use your work to write $\gcd(100, 43)$ in the form $100a + 43b$ where $a$ and $b$ are integers.

[7] 2. Let $\mathcal{S}$ be the following statement:

for all reals $n$, if $n^2$ is irrational then $n$ is irrational.

(a) Prove statement $\mathcal{S}$. Use contradiction or the contrapositive. Use only the definitions of rational and irrational.

(b) Write (as simply as possible) the *negation* of statement $\mathcal{S}$.

[10] 3. Of the following four statements, three are true and one is false. Prove the true statements and disprove the false statement. $\mathbb{Z}$ denotes the set of all integers.

(a) $\exists A \subseteq \mathbb{Z}$ so that $A - \{1\} = A - \{2\}$.

(b) $\exists A \subseteq \mathbb{Z}$ so that $A \cup \{1\} = A \cup \{2\}$.

(c) $\forall A \subseteq \mathbb{Z}\ \exists B \subseteq \mathbb{Z}$ so that $A - \{1\} = B - \{2\}$.

(d) $\forall A \subseteq \mathbb{Z}\ \exists B \subseteq \mathbb{Z}$ so that $A \cap \{1\} = B - \{2\}$.

[11] 4. Let $\mathcal{S}$ be the statement:

for all integers $a$ and $b$, if $2|a$ and $3|b$, then $6|(ab)$.

(a) Is $\mathcal{S}$ true? Give a proof or disproof.

(b) Write out (as simply as possible) the *contrapositive* of statement $\mathcal{S}$, and give a proof or disproof.

(c) Write out (as simply as possible) the *converse* of statement $\mathcal{S}$, and give a proof or disproof.

[6] 5. The sequence $x_0, x_1, x_2, \ldots$ is defined by:

$$x_0 = 1, \text{ and } x_n = 2x_{n-1} - 3n \text{ for all integers } n \geq 1.$$

Prove **using mathematical induction** that $x_n = 6 + 3n - 5(2^n)$ for all integers $n \geq 0$.

$\overline{[40]}$