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MATHEMATICS 271 MIDTERM March 9, 2006

SHOW ALL WORK. Marks for each problem are to the left of the problem number.
NO CALCULATORS PLEASE.

[4] 1. **Use the Euclidean algorithm** to find $\gcd(106, 20)$.

[7] 2. Let $\mathcal{S}$ be the following statement:

for all integers $n$, if $3n + 7$ is even then $n$ is odd.

(a) Prove statement $\mathcal{S}$. Use contradiction or the contrapositive. You may assume that every integer is either even or odd but not both, but otherwise use only the definitions of even and odd.

(b) Write out (as simply as possible) the *negation* of statement $\mathcal{S}$.

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[11] 3. Let $\mathcal{S}$ be the statement:

for all sets $A, B, C$, if $A \cap B \neq \emptyset$ and $A \cap C \neq \emptyset$, then $B \cap C \neq \emptyset$.

(a) Is $\mathcal{S}$ true? Give a proof or disproof.

(b) Write out (as simply as possible) the *converse* of statement $\mathcal{S}$, and give a proof or disproof.

(c) Write out (as simply as possible) the *contrapositive* of statement $\mathcal{S}$, and give a proof or disproof.

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[6] 4. Of the following two statements, one is true and one is false. Prove the true statement and disprove the false statement. (**Z** denotes the set of all integers.)

(a) $\forall n \in \mathbf{Z}\ \exists m \in \mathbf{Z}$ so that $3 \mid (n+m)$.

(b) $\exists n \in \mathbf{Z}$ so that $\forall m \in \mathbf{Z}$, $3 \mid (n+m)$.

[6] 5. Of the following two statements, one is true and one is false. Prove the true statement and disprove the false statement.

(a) For all sets $A$ and $B$, if $(1,2) \in A \times B$ and $(1,3) \in A \times B$, then $(2,3) \in A \times B$.

(b) For all sets $A$ and $B$, if $(1,2) \in A \times B$ and $(2,3) \in A \times B$, then $(1,3) \in A \times B$.

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[6] 6. Prove **using mathematical induction** that $2n - 1 < 3^n$ for all integers $n \geq 1$.

[40]