University of Calgary 6.1
Faculty of Science
Department of Mathematics and Statistics

Calculus 1 Fall 2009

Worksheet 5a (Answers)
[Derivatives and applications]

1. a. Determine the points on the curve

$y = 2x^3 - 9x^2 - 24x + 4$

where the tangent line to the curve is parallel to the x-axis.

The points are $(-1,17)$ *and* $(4,-108)$.

b. Determine the points on the curve

$$y = 3x^4 - 2x^3 - 9x^2 + 1$$

where the tangent line to the curve is parallel to the x-axis.

The points are $(0,1)$, $(-1,-2)$, *and* $(3/2,-173/16)$

c. Find the equation of the tangent and normal lines to the curve

$$y = x + \frac{1}{\sqrt{x}}$$

at the point where x = 4.

The tangent line is $15x - 16y = -12$;
The normal line is $32x + 30y = 263$

d. Find the equation of the tangent and normal lines to the curve

$$y = x^4 + 2x^3 + x^2 + x + 1$$

at the point where the tangent line has slope equal to
1

The tangent line has slope 1 *at the points* $(0,1)$, $(-1/2,9/16)$, *and* $(-1,0)$.

at $(0,1)$
The tangent line is $y = x + 1$
The normal line is $y = -x + 1$

at $(-1/2,9/16)$,
The tangent line is $y = x + \frac{17}{16}$
The normal line is $y = -x + \frac{1}{16}$

at $(-1,0)$
The tangent line is $y = x + 1$
The normal line is $y = -x - 1$

e. Find the equation of the tangent and normal lines to the curve

$$y = 3x^4 + 4x^3 + 12x^2 - 22x + 1$$

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at the points on the curve where the tangent line is parallel to the straight line *y = 2x - 5.*

This occurs at $(1,-2)$.

The tangent line is $y = 2x - 4$

The normal line is $2y = -x - 3$

2. Use the definition of the derivative as a limit to determine $f'(x)$ given that

a. $f(x) = \sqrt{x^2 + 4}$

b. $f(x) = \dfrac{1}{x^2 - 4}$

c. $f(x) = \dfrac{4x + 5}{3 - 5x}$

d. $f(x) = \dfrac{1}{\sqrt{4 - x^2}}$

In each case check your answer by applying the rules for differentiation.

3. In each case, determine $f'(x)$ given $f(x)$.

i. $f(x) = \sin\left(x^4 + x^3\right)$
$f'(x) = \left(4x^3 + 3x^2\right)\cos\left(x^4 + x^3\right)$

ii. $f(x) = \tan\left(\sqrt{4 - x^3}\right)$
$f'(x) = -\dfrac{3x^2}{2\sqrt{4 - x^3}}\sec^2\left(\sqrt{4 - x^3}\right)$

iii. $f(x) = \sec\left(x^{2/3} - x\right)$
$f'(x) = \left(\dfrac{2}{3}x^{-\frac{1}{3}} - 1\right)\sec\left(x^{\frac{2}{3}} - x\right)\tan\left(x^{\frac{2}{3}} - x\right)$

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iv. $f(x) = \operatorname{cosec}\left(\sqrt{x^4+1}\right)$

$$f'(x) = -\frac{2x^3}{\sqrt{x^4+1}} \operatorname{cosec}\sqrt{x^4+1}\ \cot\sqrt{x^4+1}$$

v. $f(x) = \cos(\cos(\cos x))$

$$f'(x) = [-\sin(\cos(\cos x))]\,[-\sin(\cos x)]\,[-\sin x]$$
$$= -\sin(\cos(\cos x))\cdot\sin(\cos x)\cdot\sin x$$

vi. $f(x) = \cot\left(\sin\left(x^3+1\right)\right)$

$$f'(x) = \left[-\operatorname{cosec}^2\left(\sin\left(x^3+1\right)\right)\right]\cdot\cos\left(x^3+1\right)\cdot\left(3x^2\right)$$
$$= -3x^2\operatorname{cosec}^2\left(\sin\left(x^3+1\right)\right)\cos\left(x^3+1\right)$$

vii. $f(x) = \sqrt{x^2+\sqrt{x^2+\sqrt{x^2+1}}}$

$$f'(x) = \left[\frac{1}{2}\left(x^2+\sqrt{x^2+\sqrt{x^2+1}}\right)^{-\frac{1}{2}}\right]\cdot\left[2x+\frac{1}{2}\left(x^2+\sqrt{x^2+1}\right)^{-\frac{1}{2}}\cdot\left(2x+\frac{1}{2}\left(x^2+1\right)^{-\frac{1}{2}}\cdot(2x)\right)\right]$$

4. Determine all points on each graph of the function given where the tangent line is parallel to the x-axis.

a $y = x^4 - x^2 + 1$

The points are $(0,1)$, $\left(\frac{1}{\sqrt{2}}, \frac{3}{4}\right)$, $\left(-\frac{1}{\sqrt{2}}, \frac{3}{4}\right)$

b $y = \sin 2x - 2\sin x$

The points are $\left(2k\pi+\frac{2\pi}{3}, -\sqrt{3}\right)$, $\left(2k\pi+\frac{4\pi}{3}, \sqrt{3}\right)$, $(2k\pi, 0)$ *for all integers* k.

c $y = \tan x + \cot x$

The points are $\left(k\pi+\frac{\pi}{4}, 2\right)$, $\left(k\pi+3\frac{\pi}{4}, -2\right)$ *for all integers* k.

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5. Determine whether or not the given curve has a tangent line which is parallel to the x-axis:

a $y = 2x^3 + 3x^2 + 6x + 12$

Here, $y' = 6(x^2 + x + 1)$.
since $x^2 + x + 1 = 0$ *has no real solutions,*
There is no tangent line parallel to the x*-axis.*

b $y = 2x^3 - x^2 + 2x - 1$

Here, $y' = 6x^2 - 2x + 2$.
since $6x^2 - 2x + 2 = 0$ *has no real solutions,*
There is no tangent line parallel to the x*-axis.*